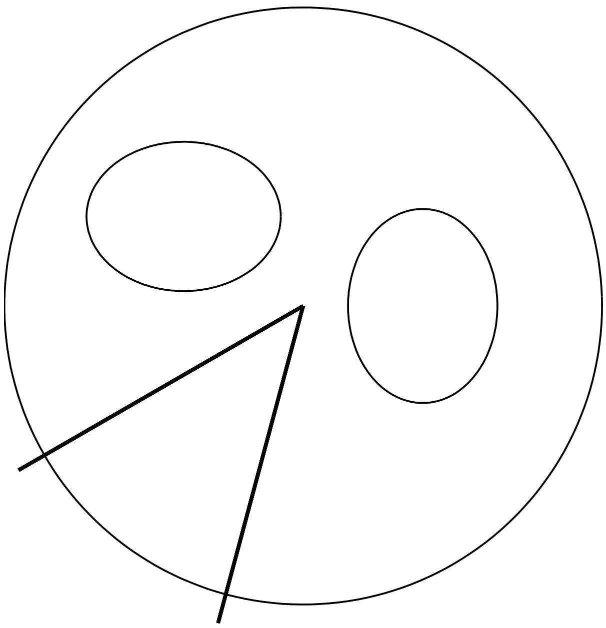
Electrical impedance tomography and Mittag-Leffler's function



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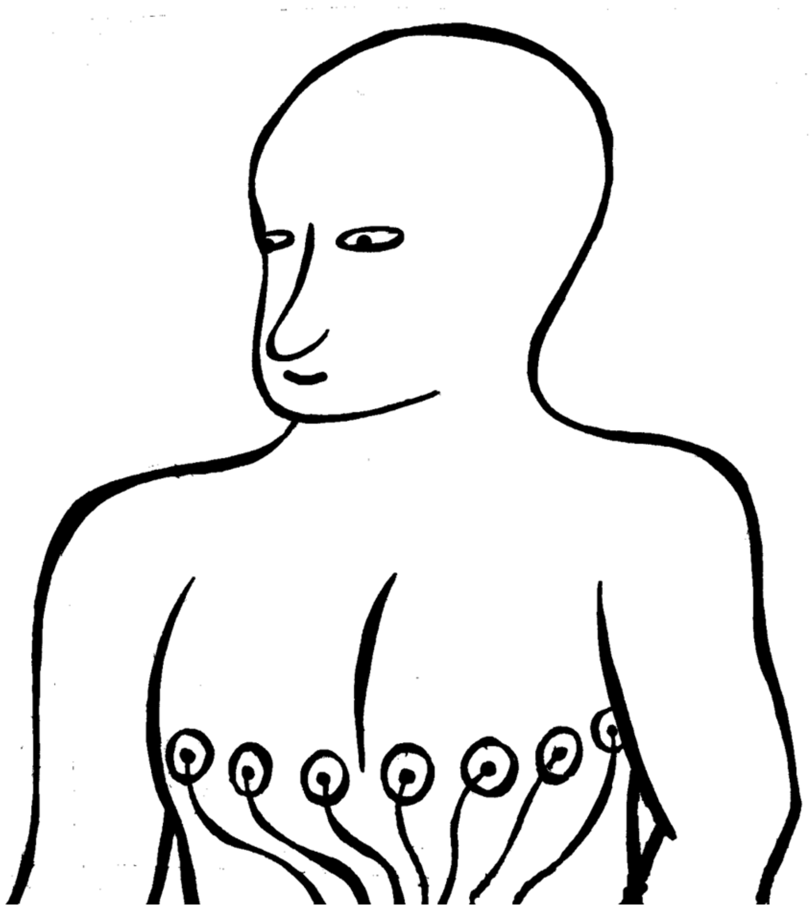
Electrical impedance tomography (EIT) is a new medical imaging technique

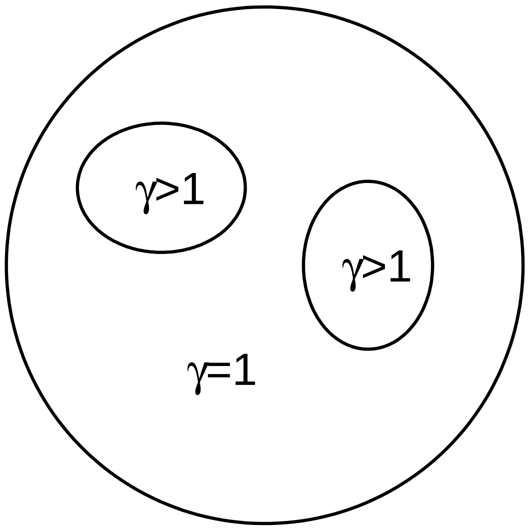


Feed electric currents through electrodes

Measure resulting voltages

Compute image of electrical conductivity in the horizontal slice

Inverse conductivity problem
is the mathematical model of EIT



$\Omega \subset \mathbb{R}^2$ is unit disc, D is inclusion with $\gamma > 1$

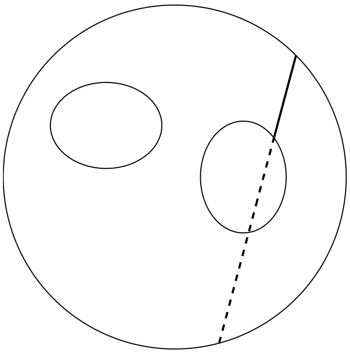
Problem in our case: Recover D

from the current-to-voltage map R_γ

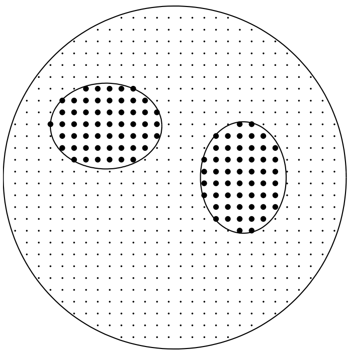
$$R_\gamma f = u|_{\partial\Omega} - \frac{1}{|\partial\Omega|}\int_{\partial\Omega} u,$$

$$\begin{cases} \nabla \cdot \gamma \nabla u = 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = f \quad \text{on } \partial\Omega. \end{cases}$$

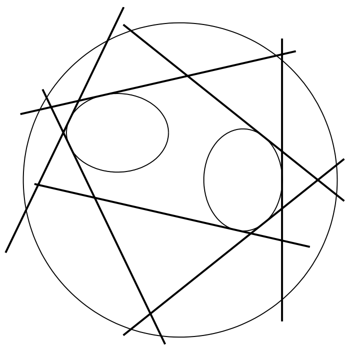
Many methods for recovering D from R_γ have been suggested



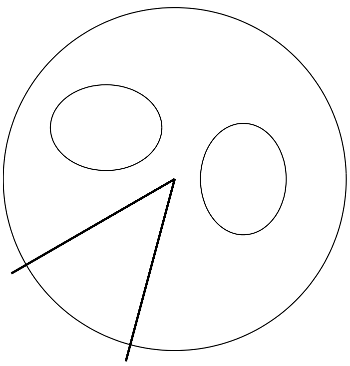
Probe method by Ikehata



Adaptation of Kirsch's method by Brühl and Hanke



Enclosure method by Ikehata, S for finding the convex hull of D



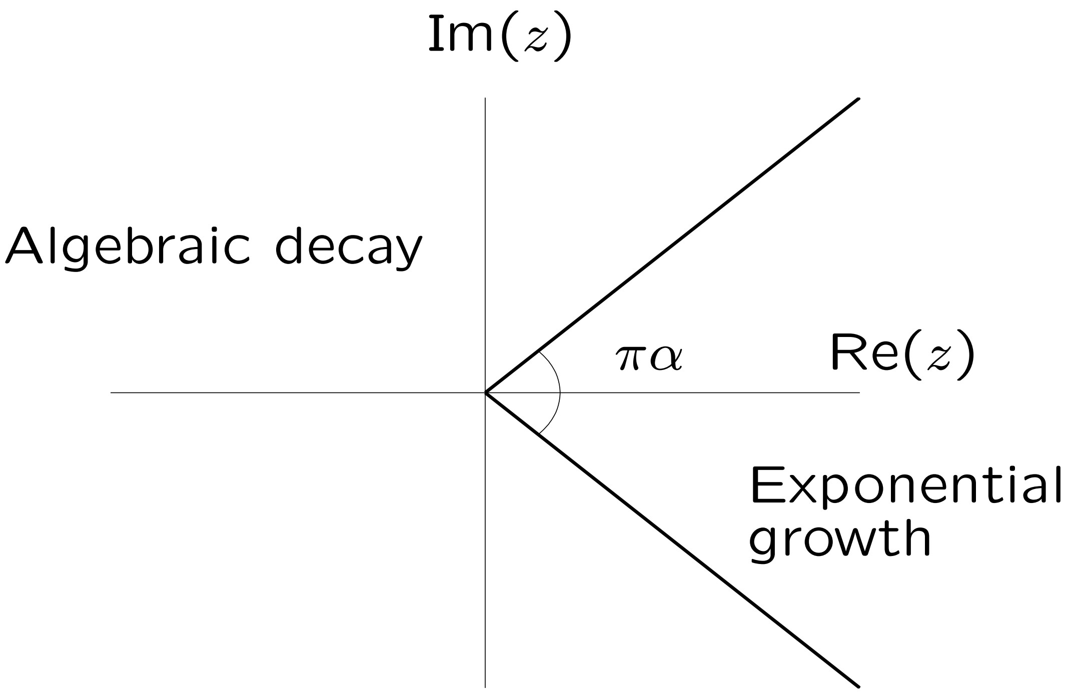
The present approach generalizes the enclosure method

Mittag-Leffler's function $E_\alpha(z)$: definition and asymptotic behavior

E_α is the entire function given by

$$E_\alpha(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(\alpha m + 1)}.$$

Asymptotic behavior of $|E_\alpha(z)|$ as $|z| \to \infty$:



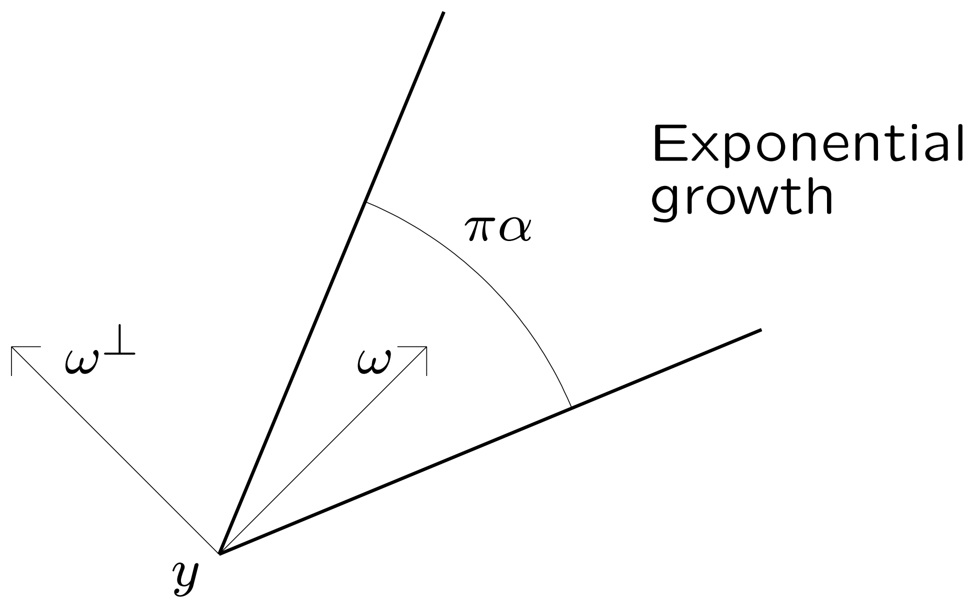
Indicator function is the key object for recovering D from R_γ

$$I^{\alpha}_{(y,\omega)}(\tau)=\int_{\partial\Omega}\overline{\frac{\partial e^{\alpha}_{\tau}}{\partial\nu}}(R_1-R_\gamma)\frac{\partial e^{\alpha}_{\tau}}{\partial\nu}d\sigma,$$

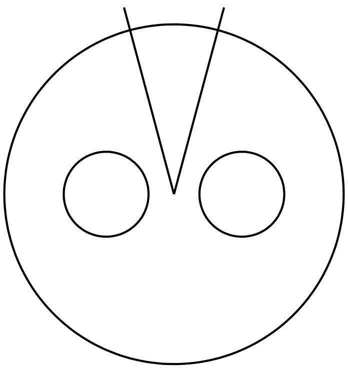
where we set for $\tau>0$

$$e^{\alpha}_{\tau}(x;y,\omega)=E_\alpha(\tau\{(x-y)\cdot\omega+i(x-y)\cdot\omega^{\perp}\}).$$

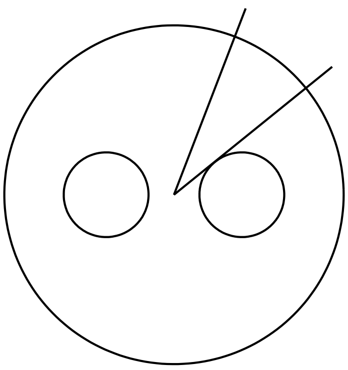
Asymptotic behavior of e^{α}_{τ} when $\tau\to\infty$:



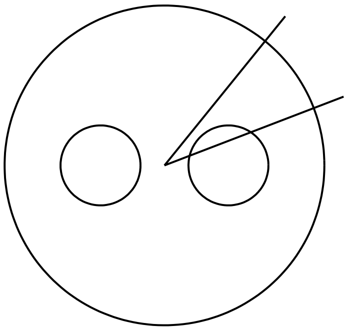
Indicator function is used to determine if a given cone intersects D



$$\lim_{\tau\to\infty} |I^{\alpha}_{(y,\omega)}(\tau)| = 0$$



$$\liminf_{\tau\to\infty} |I^{\alpha}_{(y,\omega)}(\tau)| > 0$$



$$\lim_{\tau\to\infty} |I^{\alpha}_{(y,\omega)}(\tau)| = \infty$$

[Ikehata 2001, preprint]

Reconstruction from measured data restricts the use of indicator function

$$I^{\alpha}_{(y,\omega)}(\tau) = \int_{\partial\Omega} \overline{\frac{\partial e^{\alpha}_{\tau}}{\partial \nu}} (R_1 - R_\gamma) \frac{\partial e^{\alpha}_{\tau}}{\partial \nu} d\sigma$$

In practice, R_γ is a 16×16 matrix with random error in the elements

The functions $\overline{\frac{\partial e^{\alpha}_{\tau}}{\partial \nu}}$ and $\frac{\partial e^{\alpha}_{\tau}}{\partial \nu}$ are expressed as truncated Fourier series

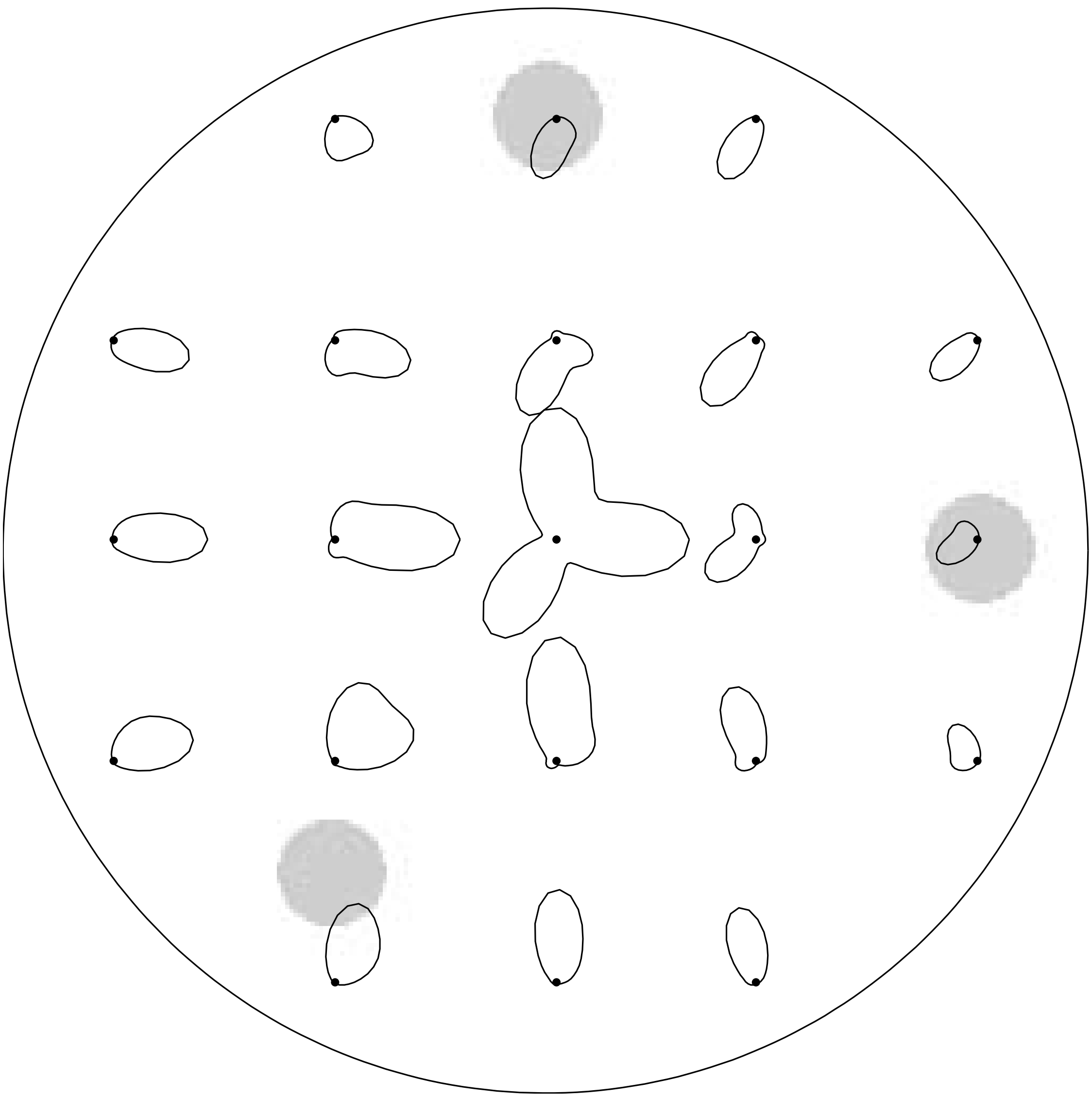
Consequently, maximum $\tau = \frac{1}{2} \neq \infty$

We recover D in three steps from the practical indicator function:

1. Exclude cones with small $|I^{1/2}_{(y,\omega)}(\frac{1}{2})|$.

2. Use local maxima to assign values to non-excluded y.

3. Recover D as a level set of the function resulting from step 2.



Three reconstructions