

## Three-dimensional X-ray imaging with few radiographs

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Workshop on Inverse Problems & Medical Imaging  
PIMS-UBC, August 6, 2003

Joint work between industry and academia:

- |                      |                              |
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| 2. Jari Kaipio       | University of Kuopio         |
| 3. Petri Koistinen   | University of Helsinki       |
| 4. Ville Kolehmainen | University of Kuopio         |
| 5. Matti Lassas      | University of Helsinki       |
| 6. Juha Pirttilä     | Invers Ltd.                  |
| 7. Samuli Siltanen   | Instrumentarium Corp.        |
| 8. Erkki Somersalo   | Helsinki Univ. of Technology |

Partially funded by the Finnish Technology Agency

## Instrumentarium Imaging



Dental X-ray imaging



Mammography



Surgical imaging

## Motivation of the study

Goal: 3D medical imaging using any X-ray source.

Problems:

- From which directions should the projection images be taken?
- How to calibrate the directions when not using a sturdy X-ray device?
- What is the minimum number of images needed?
- How to make a good enough 3-D reconstruction from few images?

## Medical applications

Sometimes 3D information on tissue is needed but a CT scan is not feasible due to cost or radiation dose.

Examples:

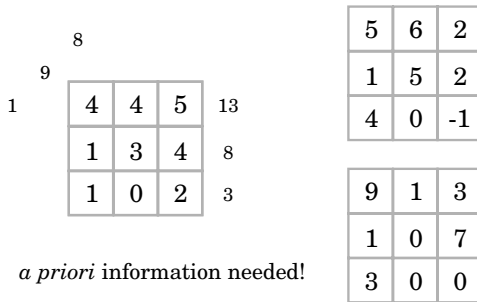
- Dental imaging
  - Implant planning
  - Safe removal of teeth
  - Detection of bone loss between teeth
- Surgical imaging
  - Position of screws inside bone
- Mammography
  - Studying locations of microcalcifications in 3D

## Intraoral dental imaging

- Digital X-ray sensor inside the mouth
- A couple of teeth visible
- Leads to limited angle tomography



## Limited angle: nonuniqueness



## Size of practical problems

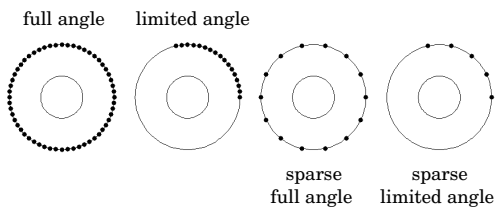
Application to 3D intraoral dental imaging:

- 42,496,000 unknowns
- 4,053,056 linear equations

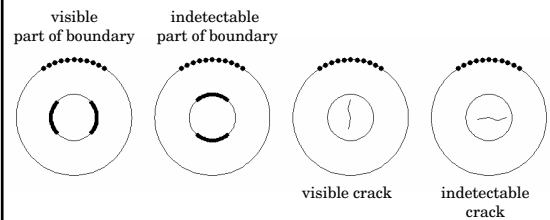
Application to 3D digital mammography:

- 4,613,734,400 unknowns
- 40,370,176 linear equations

## Sparse tomographic data



## Features visible in data



See Quinto (1993) and Ramm&Katsevich (1996)

## Bayesian inversion

- Measurement:  $m = Ax + \varepsilon$   
where matrix A models line integrals over pixels
- Bayes' formula:  $p(x|m) = \frac{p(x)p(m|x)}{p(m)} \sim p(x)p(m|x)$
- Likelihood distribution measures fitness of x to data:  
$$p(m|x) = p_\varepsilon(Ax - m) = \exp\left(-\frac{1}{2\sigma^2}\|Ax - m\|_2^2\right)$$
- Prior distribution p(x) describes *a priori* -information on the tissue
- Unknown quantity x is estimated from the posterior distribution p(x|m)

## A priori information in dental radiology

- Attenuation coefficient is non-negative (X-radiation does not intensify inside tissue)
- Different tissue types (bone, gum, pulp chamber) are approximately homogeneous
- There are clear boundaries between tissues

## Prior distribution for dental tissue

Positivity constraint

$$p_+(x) = \begin{cases} 0 & \text{if } x_j < 0 \text{ for some } j \\ 1 & \text{otherwise} \end{cases}$$

Approximate total variation penalty

$$p_{TV}(x) = \exp\left(\alpha \sum_{\text{neighbors}} |x_\ell - x_k|_\beta\right)$$

$$|t|_\beta = \frac{1}{\beta} \log(\cosh(\beta t))$$

## Computation of the MAP estimate

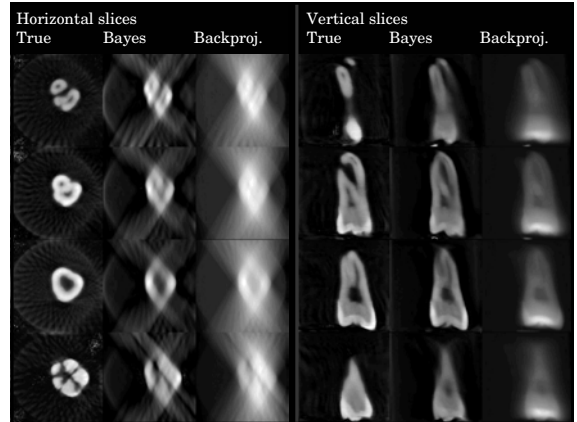
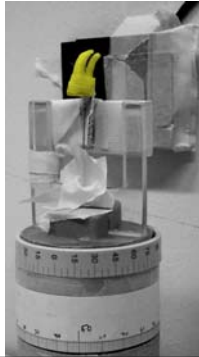
- Optimization problem:

$$x_{\text{MAP}} = \arg \min_{x_j \geq 0} \left\{ \frac{1}{2\sigma^2} \|Ax - m\|_2^2 + \alpha \sum_{\text{neighbors}} |x_\ell - x_k|_\beta \right\}$$

- Equivalent to total variation regularization
- Great dimension is challenging
  - Barzilai-Borwein gradient method used

## First experiment: tooth

- 23 projections from full angle (187 deg)
- 9 projections from limited angle (68 deg)
- Comparison between full angle and limited angle reconstructions
- Comparison between Bayesian inversion and (unfiltered) backprojection

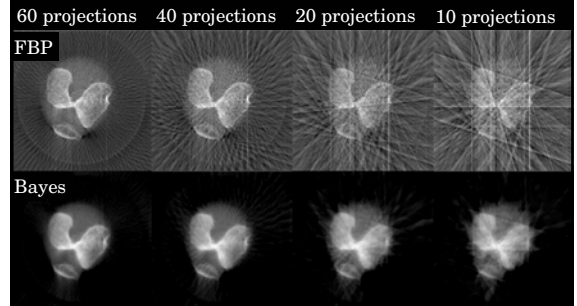


## Second experiment: knee

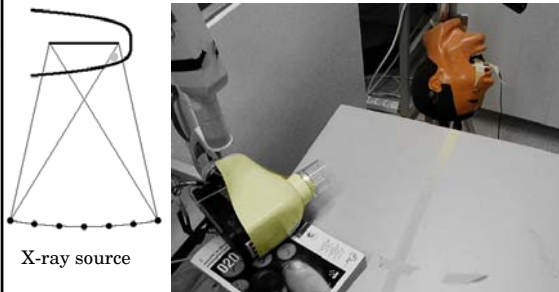
- A human knee phantom was imaged from full angle with a surgical C-arm X-ray device
- Reconstruction with few projections from full angle
- Comparison between Bayesian inversion and filtered backprojection



## Comparison



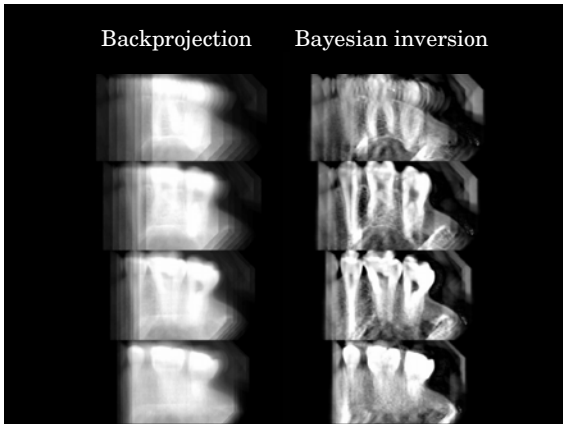
### Third experiment: intraoral dental imaging



### Third experiment: intraoral dental imaging

- Digital intraoral sensor was placed inside the mouth of a skull phantom
- Seven radiographs were taken from an opening angle of 60 degrees
- Projection directions were calibrated using a metal ball attached to teeth
- Leads to a combination of limited-angle and local tomography
- Comparison between Bayesian inversion and (unfiltered) backprojection

Backprojection      Bayesian inversion



### Conclusion

- We discussed a new medical imaging modality: 3D X-ray imaging with few radiographs
- Features of the new method:
  - Low radiation dose
  - Cost-effective: any X-ray source becomes a 3-D imaging device
- Problems in the new method:
  - Leads to ill-posed reconstruction problems
  - Projection directions must be chosen carefully
- Bayesian inversion promising for reconstruction
  - allows effective use of *a priori* information

### Future challenges

- Designing new priors
  - *A priori* information is typically available, but how to express it mathematically?
- Speeding up algorithms
  - Optimization methods
  - MCMC algorithms
- Clinical trials
  - Find out end users' real needs
  - Learn acceptable computation times

### References

- Siltanen et al. 2003,  
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- Kolehmainen et al. 2003,  
*Statistical inversion for medical X-ray tomography with few radiographs II: Applications to dental radiology*,  
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