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Modelling and analysing oriented fibrous structures

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Abstract. A mathematical model for fibrous structures using a direction dependent scaling law is presented. The orientation of fibrous nets (e.g. paper) is analysed with a method based on the curvelet transform. The curvelet-based orientation analysis has been tested successfully on real data from paper samples: the major directions of fibrefibre orientation can apparently be recovered. Similar results are achieved in tests on data simulated by the new model, allowing a comparison with ground truth.

1. Introduction

The quality of paper and also other fibre-based products depends essentially on how wood fibres are distributed in a more or less random network of predominantly planar orientation. For this reason, it would be important to be able to measure and control orientation and other properties of the fibre network already during the manufacturing process.

The formation of paper has traditionally been inspected visually and later by analysing in different ways its optical transmission image [1]. However, there are artefacts in optical transmission images due to strong scattering of visible electromagnetic radiation in paper-like fibrous structures. Also, it is not certain as yet from which part of the paper structure an optical transmission image contains information. Therefore, it is of utmost importance to properly calibrate the information gained from optical images. There are two especially suitable methods of calibration: x-ray tomography and simulated networks with precisely known properties.

A method to transform an optical transmission image to one that fairly closely resembles that of x-ray transmission has already been formulated [2]. In this work we present a mathematical model for simulating simple fibre nets using a direction dependent scaling law. The orientation of simulated data is analysed with a method based on the curvelet transform [3, 4], which has been successfully tested in analysing real data from paper samples [5]. The aim of the work is to design an algorithm that simulates a fibrous system from a given orientation distribution.

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2. Theory

2.1. Analysing orientation with curvelets

The orientation of complex patterns has traditionally been analysed by applying the Fourier transform [6] or gradient-based methods like the structure tensors [7, 8]. However, more sophisticated methods, especially the wavelet transform, [9] have become popular in the last two decades. Furthermore, in recent years transforms like the curvelet, contourlet and shearlet transform have been developed and proved to be well-suited for some applications [3, 4, 10, 11].

The curvelet transform is tightly localised in both space and frequency domain, and has in addition an angle parameter that makes it an optimal tool for orientation analysis. The mother curvelet is defined at each scale $0 < a < a_0$ in the frequency domain as

$$\widehat{\gamma}_{a00}(r\cos(\omega), r\sin(\omega)) = a^{\frac{3}{4}} W(ar)V(\omega/\sqrt{a}),$$

where $r \leq 0$ and $0 \leq \omega \leq 2\pi$, the radial window W is a non-negative, infinitely smooth real-valued function supported inside the interval $(\frac{1}{2}, 2)$, and the angular window V is a non-negative, infinitely smooth real-valued function supported on the interval $[-1, 1]$ (see Figure 1). The whole curvelet is achieved as

$$\gamma_{ab\theta}(x) = \gamma_{a00}(R_{-\theta}(x - b)),$$

where $0 \leq \theta \leq 2\pi$ is a rotation parameter, $b \in \mathbb{R}^2$ is a translation parameter, and R_θ is the matrix of planar counter-clockwise rotation by the angle θ. The curvelet transform is then defined by

$$\Gamma_f(a, b, \theta) := \langle \gamma_{ab\theta}, f \rangle = \int_{\mathbb{R}^2} f(x)\gamma_{ab\theta}(x)\, dx,$$

for all $0 < a < a_0$, $b \in \mathbb{R}^2$ and $0 \leq \theta \leq 2\pi$.

Curvelets follow the parabolic scaling law in the aspect ratio of the area that contains most of their energy. If we take a piece of smooth curve (corresponding to an edge of a fibre) with a length of about a, then the whole piece will fit into a rectangle with the side lengths a and $\sqrt{a}$. We can think of $\gamma_{ab\theta}$ as a sensor that tries to detect if there is a fibre with orientation θ in the neighbourhood of b. If f denotes a fibrous image, then the inner product $\langle \gamma_{ab\theta}, f \rangle$ presents the response of sensor $\gamma_{ab\theta}$. A small value of parameter a means we focus into a part of a fibre, while its larger values can embed a whole fibre. If there is no fibre with the orientation angle θ located at point b, the value of $|\langle \gamma_{ab\theta}, f \rangle|$ is very small.

Figure 1. Schematic illustration of curvelet functions, left: curvelet functions of one angle θ in frequency domain, right: the same curvelet functions in spatial domain.

2.2. The H model for fibrous systems

The statistics of physical parameters (e.g. density) of fibrous structures measured on lines depend on the line direction. If we assume that the physical parameters on a line follow the fractional

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Brownian motion statistics, then the Hurst index models the order of the fractional Brownian motion [12]. We create a random model for objects of controllable lengths, where a greater value of the Hurst index means larger features in the object.

Assume now that $Y(x)$ is a stationary two-dimensional Gaussian random field with zero mean, which satisfies a non-standard scaling law for the covariance operator. Here, a Gaussian random field $Y(x)$ is stationary if the covariance function $K_Y(x,y) = \mathbb{E}\,(Y(x)Y(y))$ satisfies $K_Y(x+a, y+a) = K_Y(x,y)$. We consider two one-dimensional stationary Gaussian processes p_1 and p_2 having covariances

$$\mathbb{E}\,(p_1(t)p_1(s)) \sim 1 - |t-s|^{2H_1} \quad \text{and} \quad \mathbb{E}\,(p_2(t)p_2(s)) \sim 1 - |t-s|^{2H_2} \quad \text{as } t \to s,$$

and zero mean, $\mathbb{E}\,p_j = 0$. This means that p_1 and p_2 are independent fractional Brownian motions with Hurst indices H_1 and H_2. The random field $Y(x)$ is obtained by taking oriented fibres having one end point at (y_1, y_2), i.e. the functions $(x_1, x_2) \mapsto K_{y_1,y_2}(x_1,x_2) = k_1(x_1-y_1)k_2(x_2-y_2)$, and summing such functions with all possible (y_1, y_2) together with random weights

$$Y(x_1,x_2) = \int_{\mathbb{R}^2} k_1(x_1-y_1)k_2(x_2-y_2)W(y_1,y_2)\,dy_1dy_2,$$

where $W(y_1,y_2)$ is a 2D Gaussian white noise. The Fourier transform of the function $\mathcal{K}(x_1,x_2) = k_1(x_1)k_2(x_2)$ satisfies

$$\widehat{\mathcal{K}}(\xi_1,\xi_2) = \widehat{k}_1(\xi_1)\widehat{k}_2(\xi_2) \sim \quad c_1c_2(1+|\xi_1|)^{-1/2-H_1}(1+|\xi_2|)^{-1/2-H_2} \quad \text{as } \xi \to \infty.$$

3. Results and discussion

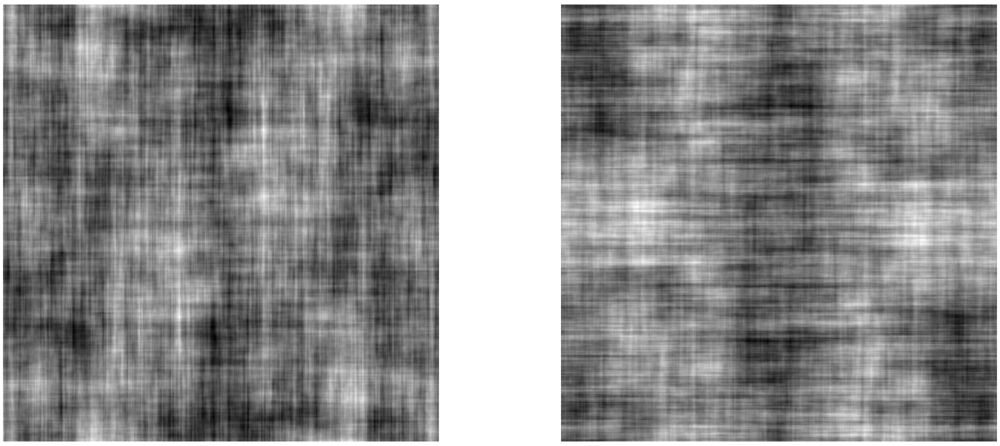

Figure 2. Left: fibre net with Hurst indices $H_1 = 0.3$ and $H_2 = 0.5$, right: fibre net with Hurst indices $H_1 = 0.5$ and $H_2 = 0.3$.

The H model gives us fibre nets with two perpendicular orientations. Two samples of fibre nets simulated with the model are shown in Figure 2. A common example of this type of fibre net is newsprint. The simulated fibre nets can then be transformed to change the orientation of the fibres. Examples of the transformed fibre nets are shown in Figure 3. Note that the vertical fibres are turned to the same orientation in both transformations, but the horizontal fibres are transformed differently in rotation and shearing. The orientation analysis of these fibre nets is

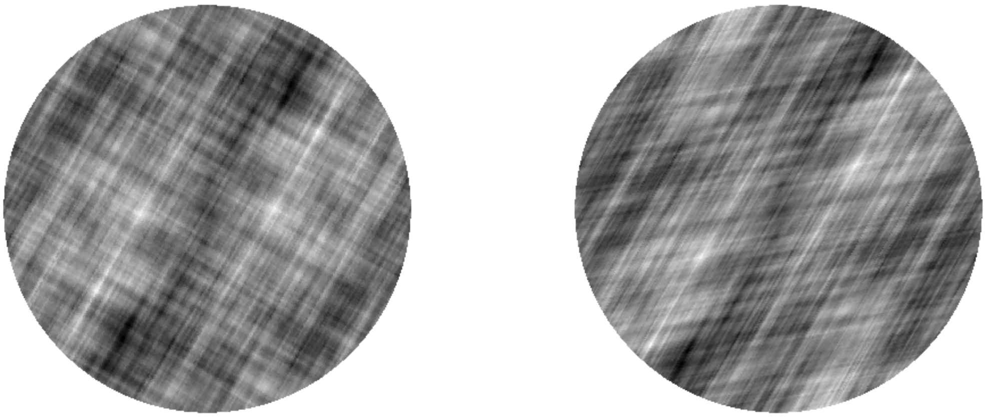

Figure 3. Left: the left fibre net of Figure 2 rotated 30° clockwise, right: the same fibre net [illegible]

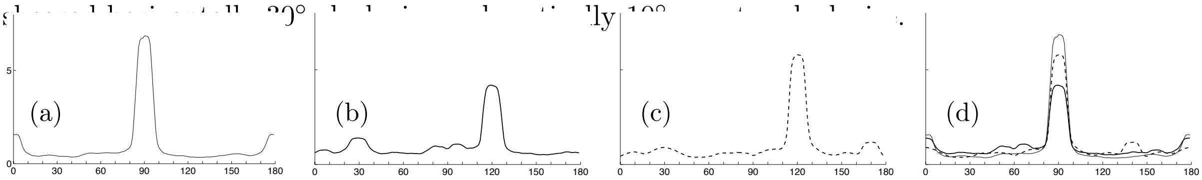

Figure 4. (a): orientation analysis of the left fibre net of Figure 2, (b): orientation analysis of the left rotated fibre net of Figure 3, (c): orientation analysis of the right sheared fibre net of Figure 3, (d): orientation curves (a)-(c) shifted to match the strongest orientation.

shown in Figure 4. The strongest orientations are found in the expected positions, although the transformations vary the orientation strengths.

The numerical evidence shows that we get the desired orientations in the fibre nets simulated with the H model. A further goal is to analyse the orientations in real fibre data and then simulate data with the exact same orientations.

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