Classifying Stroke Using Electrical Impedance Tomography

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Links to open computational resources

Open EIT datasets:
- Finnish Inverse Problems Society (FIPS) dataset page

Reconstruction algorithms: FIPS Computational Blog
- The D-bar Method for EIT—Simulated Data
- The D-bar Method for EIT—Experimental Data

For these slides see: http://www.siltanen-research.net/talks.html
Outline

Electrical impedance tomography (EIT)

Complex geometric optics (CGO) solutions, D-bar method

Application of EIT to stroke

Virtual Hybrid Edge Detection (VHED)
This section concentrates on applications of EIT to chest imaging

Medical applications: monitoring cardiac activity, lung function, and pulmonary perfusion. Also, electrocardiography (ECG) can be enhanced using knowledge about conductivity distribution.
The D-bar method works for real EIT data, such as laboratory phantoms and *in vivo* human data.

Saline and agar phantom

Reconstruction \((R = 4)\)

[Isaacson, Mueller, Newell & S 2004]
[Montoya 2012]
The mathematical model of EIT is the inverse conductivity problem introduced by Calderón

Let $\Omega \subset \mathbb{R}^2$ be the unit disc and let conductivity $\sigma : \Omega \to \mathbb{R}$ satisfy

$$0 < M^{-1} \leq \sigma(z) \leq M.$$ 

Applying voltage $f$ at the boundary $\partial \Omega$ leads to the elliptic PDE

$$\begin{cases}
\nabla \cdot \sigma \nabla u = 0 \text{ in } \Omega, \\
u|_{\partial \Omega} = f.
\end{cases}$$

Boundary measurements are modelled by the Dirichlet-to-Neumann map

$$\Lambda_\sigma : f \mapsto \sigma \frac{\partial u}{\partial \vec{n}}|_{\partial \Omega}.$$ 

Calderón’s problem is to recover $\sigma$ from the knowledge of $\Lambda_\sigma$. It is a nonlinear and ill-posed inverse problem.
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There exists a nonlinear Fourier transform adapted to electrical impedance tomography.
The nonlinear Fourier transform can be recovered from infinite-precision EIT measurements

$\Lambda_\sigma$ BIE

Ideal measurement

Nonlinear IFFT

[Nachman 1996]
Measurement noise prevents the recovery of the nonlinear Fourier transform at high frequencies.
We truncate away the bad part in the transform; this is a nonlinear low-pass filter.
The D-bar method is a regularization strategy for reconstructing the full conductivity distribution.

Practical measurement

- BIE
- Lowpass
- Nonlinear IFFT

[S, Mueller & Isaacson 2000]
[Knudsen, Lassas, Mueller & S 2009]
D-bar images can be sharpened by Deep Learning

[Hamilton & Hauptmann 2017]
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Motivation of this study: imaging stroke with EIT

Ischemic stroke: low conductivity.
CT image from Jansen 2008

Hemorrhagic stroke: high conductivity.
CT image from Nakano et al. 2001

Same symptoms in both cases!

Difficulties: resistive skull layer and unknown background. However, see
- Romsauerova, McEwan, Horesh, Yerworth, Bayford & Holder 2006
- Shi, You, Xu, Wang, Fu, Liu & Dong 2009
- Bayford & Tizzard 2012
- Malone, Jehl, Arridge, Betcke & Holder 2014
- Boverman, Kao, Wang, Ashe, Davenport & Amm 2016
Brain EIT imaging is based on covering the head partly by electrodes

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Ωcm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cortex</td>
<td>229</td>
</tr>
<tr>
<td>White matter</td>
<td>344</td>
</tr>
<tr>
<td>Blood</td>
<td>125</td>
</tr>
<tr>
<td>CS fluid</td>
<td>69</td>
</tr>
<tr>
<td>Scalp</td>
<td>490</td>
</tr>
<tr>
<td>Skull</td>
<td>6500</td>
</tr>
</tbody>
</table>

The current activity was initiated by Alex Ross from GE. He is a former student of David Isaacson.
We have a collaboration network in place for the stroke-EIT project

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The results in this talk are a joint work with

**Allan Greenleaf**, University of Rochester, NY, USA

**Matti Lassas**, University of Helsinki, Finland

**Minh Mach**, University of Helsinki, Finland

**Matteo Santacesaria**, University of Helsinki, Finland

**Gunther Uhlmann**, University of Washington, USA

**Toshiaki Yachimura**, Tohoku University, Japan
We consider exponentially behaving Complex Geometric Optics (CGO) solutions

Denote \( x = (x_1, x_2) \in \mathbb{R}^2 \) and \( k = i\tau \theta \) where

\[
\theta = \theta_1 + i\theta_2 \in \mathbb{C} \quad \text{with} \quad |\theta| = 1.
\]

Let \( z = x_1 + ix_2 \in \mathbb{C} \) and

\[
\eta = \eta_R + i\eta_I = (\theta_1 + i\theta_2, -\theta_2 + i\theta_1) \in \mathbb{C}^2,
\]

so that \( z\theta = x_1\theta_1 - x_2\theta_2 + i(x_1\theta_2 + x_2\theta_1) = x \cdot \eta \). Note that \( \eta \cdot \eta = 0 \). We consider solutions of the conductivity equation

\[
\nabla \cdot \sigma \nabla u = 0 \quad \text{in} \ \Omega,
\]

with a strictly positive conductivity \( \sigma \in L^\infty(\Omega) \), of the form

\[
u(x) = e^{i\tau \theta z} w(x, \tau) = e^{i\tau \eta \cdot x} w(x, \tau).\]
Since $u(x) = e^{i\tau \eta \cdot x} w(x, \tau)$ satisfies the conductivity equation,

\[ 0 = \frac{1}{\sigma(x)} \nabla \cdot (\sigma(x) \nabla u(x)) \]

\[ = (\Delta + \frac{1}{\sigma}(\nabla \sigma) \cdot \nabla)(e^{i\tau \eta \cdot x} w(x, \tau)) \]

\[ = \left( \Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + (\frac{1}{\sigma} \nabla \sigma) \cdot (\nabla + i\tau \eta) w(x, \tau) \right) e^{i\tau \eta \cdot x}. \]

Hence, we have

\[ \Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + (\frac{1}{\sigma} \nabla \sigma) \cdot (\nabla + i\tau \eta) w(x, \tau) = 0. \]
New trick: apply one-dimensional Fourier transform to the complex spectral parameter

Let \( \hat{w}(x, t) \) be the Fourier transform of \( w(x, \tau) \) in the \( \tau \) variable:

\[
\hat{w}(x, t) = \mathcal{F}w(x, t) = \int_{\mathbb{R}} e^{-it\tau} w(x, \tau) \, d\tau.
\]

We call \( t \) the \textit{pseudo-time} corresponding to complex frequency \( \tau \). Then equation

\[
\Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot \left( \nabla + i\tau \eta \right) w(x, \tau) = 0
\]

yields

\[
\Delta \hat{w}(x, t) + 2\eta \frac{\partial}{\partial t} \cdot \nabla \hat{w}(x, t) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot \left( \nabla + \eta \frac{\partial}{\partial t} \right) \hat{w}(x, t) = 0.
\]
Complex principal type operator leads to singularities propagating along leaves

In the equation

\[ \Delta \hat{w}(x, t) - 2 \eta \frac{\partial}{\partial t} \cdot \nabla \hat{w}(x, t) + \frac{1}{\sigma} (\nabla \sigma) \cdot (\nabla - \eta \frac{\partial}{\partial t}) \hat{w}(x, t) = 0 \]

the principal part is

\[ \Delta - 2 \eta \frac{\partial}{\partial t} \cdot \nabla, \]

which is a complex principal type operator in the sense of Duistermaat and Hörmander.

For a real principal type operator the characteristic singularities propagate along one-dimensional rays. For instance, for the wave equation the light-like singularities propagate along light rays.

For a complex principal type operator the characteristic singularities propagate along two-dimensional surfaces called leaves.
Let us consider a rotationally symmetric conductivity with jump along the circle $|z| = 0.2$. 

$$k = \tau e^{i\varphi} \quad \varphi = 0$$
We use propagation and reflection of singularities along leaves for detecting inclusions.

Here the magenta plane wave hits the blue surface, producing light blue reflected waves.
Here are the two lowest-order odd terms in the scattering series, with subtraction

\[ [T^+_1 - T^-_1] \mu \]

\[ [T^+_3 - T^-_3] \mu \]
We can see the difference in conductivity reflected in the VHED projections (blue and red graphs).
Given unrealistic-precision EIT measurements on full boundary we can classify the stroke easily.
Practical EIT measurements blur the information due to heavily windowed Fourier transform
Simulations by Antti Hannukainen and Minh Mach
Practical problems in applying VHED

VHED works with ideal simulated data and simple digital phantoms. However, these issues must be solved before it can be applied to stroke classification:

Data is noisy. We know the Fourier transform of the desired function only in an interval $[-R, R]$ with $R \approx 4$.

Anatomy is complicated. Need to be tested with realistic phantoms.

We can only measure on a part of the boundary. Some progress is reported in [Hauptmann, Santacesaria and S 2017].

Measurements are done using a finite number of electrodes. Recovering CGO solutions from electrode data needs new research.

People are three-dimensional. VHED needs to be extended to 3D.
Thank you for your attention!
This project started in 2000