Sparse-data tomographic imaging in practice

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Seminar talk at Colorado State University
Fort Collins, CO, USA, March 7, 2019
Finnish Centre of Excellence in Inverse Modelling and Imaging
2018-2025

Finland

• Oulu University
• University of Eastern Finland
• University of Jyväskylä
• Lappeenranta University of Technology
• Tampere University of Technology
• Finnish Meteorological Institute

Aalto University
We can see through a box of candy!

https://www.dropbox.com/s/e7i3exqc4sdpr1s/Sisu2.mp4?dl=0
Here is a 2D slice through a human head

Andrew Ciscel, Wikimedia commons
Now the attenuation process is more complicated because there are different tissues

https://youtu.be/lvUAOeS1sv8
After calibration we are observing how much attenuating matter the X-ray encounters in total

https://youtu.be/RFArLtWEfsQ
This sweeping movement is the data collection mode of first-generation CT scanners

https://youtu.be/JHUz5oyeZb0
Data is collected by rotating the system around the patient

https://youtu.be/newxZbw7YAs
Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography.

Hounsfield (top) and Cormack received Nobel prizes in 1979.
Modern CT scanners look like this
Modern scanners rotate at high speed

https://commons.wikimedia.org/wiki/File:CT-Rotation.ogv
This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAql
This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58IO1FKw
Diagnosing stroke with X-ray tomography

Ischemic stroke

CT image from Jansen 2008

Hemorrhagic stroke

CT image from Nakano et al. 2001
Outline
We collected X-ray projection data of a walnut from 1200 directions

Data collection: thanks to Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

FBP with comprehensive data (1200 projections)

FBP with sparse data (20 projections)
Sparse-data reconstruction of the walnut using non-negative total variation regularization

\[
\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}
\]
Outline
Let us construct a digital phantom for tomography.
Discretize the unknown by dividing it into pixels

Target (unknown)  

32×32 pixel grid
System matrix $A$, given by the grid and X-rays

$735 \times 1024$ system matrix $A$, only nonzero elements shown

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System matrix $A$, given by the grid and X-rays

$735 \times 1024$ system matrix $A$, only nonzero elements shown

$32 \times 32$ pixel grid
What can we expect to see from sparse data?

Object $\rightarrow A \rightarrow$ Sinogram

**Theorem 4.2.** A finite set of radiographs tells nothing at all.

For some reason this theorem provokes merriment. It is so plainly one of those mathematical ideals untainted by any possibility of practical application.

[Cormack 1963], [Smith, Solmon & Wagner 1977, Theorem 4.2]
Naive reconstruction using the minimum norm solution from the normal equation $(A^T A) f^† = A^T m$

Original phantom, values between zero (black) and 0.44

Reconstruction: minimum pixel value $-1.5 \cdot 10^{14}$, maximum value $1.3 \cdot 10^{14}$
Naive reconstruction using the minimum norm solution with non-negativity constraint

Original phantom, values between zero (black) and 0.44

Reconstruction: minimum value 0, maximum value 2.3
Illustration of the ill-posedness of sparse tomography

Difference 0.00992
Illustration of the ill-posedness of sparse tomography

Difference 0.00983
Singular value decomposition $A = U^T D V$

735×1024 system matrix $A$, only nonzero elements shown

Singular values of $A$ (diagonal of $D$)
Outline
Daubechies, Defrise and de Mol introduced a revolutionary inversion method in 2004

Consider the sparsity-promoting variational regularization

$$\arg\min_{f \in \mathbb{R}^n} \left\{ \| Af - m \|_2^2 + \mu \| Wf \|_1 \right\},$$

where $W$ is an orthonormal wavelet transform. The minimizer can be computed using the iteration

$$f_{j+1} = W^{-1} S_\mu W \left( f_j + A^T (m - Af_j) \right),$$

where the soft-thresholding operation

$$S_\mu(x) = \begin{cases} 
  x + \frac{\mu}{2} & \text{if } x \leq -\frac{\mu}{2}, \\
  0 & \text{if } |x| < \frac{\mu}{2}, \\
  x - \frac{\mu}{2} & \text{if } x \geq \frac{\mu}{2}, 
\end{cases}$$

is applied to each wavelet coefficient separately.
We modify the method so that non-negativity constraint has rigorous mathematical foundation

The minimizer

$$\argmin_{f \in \mathbb{R}^n_+} \left\{ \frac{1}{2} \| Af - m \|_2^2 + \mu \| Wf \|_1 \right\}$$

can be computed using this iteration:

\[
y^{(i+1)} = \mathbb{P}_{C} \left( f^{(i)} - \tau \nabla g(f^{(i)}) - \lambda W^T v^{(i)} \right)
\]

\[
v^{(i+1)} = \left( I - S_{\mu} \right) \left( Wy^{(i+1)} + v^{(i)} \right)
\]

\[
f^{(i+1)} = \mathbb{P}_{C} \left( f^{(i)} - \tau \nabla g(f^{(i)}) - \lambda W^T v^{(i+1)} \right)
\]

where \( \tau > 0, \lambda > 0 \) and \( g(f) = \frac{1}{2} \| Af - m \|_2^2 \). Here \( \mathbb{P}_{C} \) denotes projection to the non-negative “quadrant.”

[Loris & Verhoeven 2011], [Chen, Huang & Zhang 2016]
Illustration of the Haar wavelet transform
Sparse-data reconstruction of the walnut using Haar wavelet sparsity

Filtered back-projection

Constrained Besov regularization

$$\arg \min_{f \in \mathbb{R}^n} \left\{ \|Af - m\|_2^2 + \alpha \|f\|_{B^{1}_{11}} \right\}$$
How to choose the thresholding parameter $\mu$? Here it is too small.
How to choose the thresholding parameter $\mu$? Here it is too large.
Automatic parameter choice using controlled wavelet-domain sparsity (CWDS)

Assume given the \textit{a priori} sparsity level $0 \leq C_{pr} \leq 1$. Denote by $C_j$ the sparsity of the $j$th iterate $f_j \in \mathbb{R}^n$:

$$C_j = \frac{\text{(number of nonzero elements in } Wf_j \in \mathbb{R}^n)}{n}.$$ 

The CWDS iteration is based on proportional-integral-derivative (PID) controllers:

$$\mu^{(i+1)} = \mu^{(i)} + \beta (C^{(i)} - C_{pr}).$$

[Purisha, Rimpeläinen, Bubba & S 2018]
CWDS choice of the thresholding parameter $\mu$
CWDS choice of the thresholding parameter $\mu$
Shearlet coefficients at coarse scale $1/8$

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 2/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 3/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 4/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 5/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 6/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 7/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at coarse scale 8/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at medium scale 1/8

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
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Shearlet coefficients at medium scale 3/8

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Shearlet coefficients at medium scale 5/8

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Shearlet coefficients at fine scale 1/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale $2/16$

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale 3/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale 5/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
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Shearlet coefficients at fine scale 11/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale 12/16.

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale 13/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
Shearlet coefficients at fine scale 15/16

We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
We use Shearlab [Kutyniok, Shahram & Zhuang 2012].
The shearlet transform gives multi-resolution and orientation-aware building blocks for image data.

Schematic diagram of the frequency plane tiling of several elements of a 2D shearlet system, for different values of dilation and shearing parameters.
This is a joint work with

**Tatiana Bubba**, University of Helsinki, Finland

**Sakari Karhula**, Oulu University Hospital, Finland

**Juuso Ketola**, Oulu University Hospital, Finland

**Maximilian März**, TU Berlin

**Miika T. Nieminen**, University of Oulu, Finland

**Zenith Purisha**, University of Helsinki, Finland

**Juho Rimpeläinen**, University of Helsinki, Finland

**Simo Saarakkala**, Oulu University Hospital, Finland
We consider small specimens of human bone imaged using microtomography.

Slice of 3D reconstruction by FDK based on 596 angles.

Three-dimensional structure.
We pick out a smaller region of interest for osteoarthritis analysis.

Slice of 3D reconstruction by FDK based on 596 angles

Slice of 3D region of interest after binary thresholding
We use two numerical quality measures applied to segmented three-dimensional bone structure:

- **Trabecular thickness**
- **Trabecular separation**

[Bouxsein, Boyd, Christiansen, Guldberg, Jepsen, & Müller 2010]
The goal is to reduce measurement time by recording fewer radiographs.

3D FDK reconstruction based on 40 angles

3D shearlet-sparsity reconstruction based on 40 angles
Bone quality parameters from ground truth

Projections: 300

Thickness: 0.34
Separation: 0.71

Projections: 300

Thickness: 0.37
Separation: 0.35

[Purisha et al. 2019]
Results from FDK reconstructions

Purisha et al. 2019
Results from 3D shearlet-sparsity reconstructions

[Purisha et al. 2019]
Application: dental implant planning, where a missing tooth is replaced with an implant
This is the classical imaging procedure of the panoramic X-ray device.
The resulting image shows a sharp layer positioned inside the dental arc.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics.

A panoramic dental image offers a general overview showing all teeth and other structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
We reprogram the panoramic X-ray device so that it collects projection data by scanning
We reprogram the panoramic X-ray device so that it collects projection data by scanning

Number of projection images: 11

Angle of view: 40 degrees

Image size: $1000 \times 1000$ pixels

The unknown vector $f$ has 7,000,000 elements.
Standard Cone Beam CT reconstruction delivers 100 times more radiation than VT imaging

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke 2006
Kolehmainen, Lassas & S 2008
Cederlund, Kalke & Welander 2009
Hyvönen, Kalke, Lassas, Setälä & S 2010
U.S. patent 7269241, thousands of VT units in use
The VT device was developed in 2001–2012 by

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
This part is a joint work with

Alexander Meaney, University of Helsinki, Finland

Esa Niemi, Eniram Ltd., Finland

Aaro Salosensaari, University of Helsinki, Finland

Industrial partners:

Kemppi Ltd. (welding tool manufacturer)

Ajat Ltd. (X-ray detector manufacturer)
Two steel pipes partly welded together
This is the limited-angle measurement geometry for a narrow CaTd direct conversion detector.
Reconstruction algorithm: variant of TVR-DART

With a regularization parameter $\alpha > 0$, we minimize

$$\arg \min_{x \in \mathbb{R}^N} \{ \|AS(x) - m\|_2^2 + \alpha TV_{\beta}(x) \},$$

where $S : \mathbb{R}^N \to \mathbb{R}^N$ is a soft segmentation function given by

$$S(x) = \sum_{g=2}^{G} (\rho_g - \rho_{g-1}) u(x - \tau_g, k_g),$$

with $u(x, k_g) = (1 + e^{-2k_g x})^{-1}$, and $k_g = K / (\rho_g - \rho_{g-1})$. Here $G = 2$ is the number of materials and $3 < K < 6$ is called a transition constant. The parameters $\rho_g$ are the pre-known attenuation values of the materials and $\tau_g$ are the threshold levels between the different attenuations with $\tau_1 = 0$. Above $TV_{\beta}$ is

$$TV_{\beta} = \sum_{i,j} \sqrt{(x_{i+1} - x_i)^2 + (x_{i+n} - x_i)^2 + \beta}, \quad \beta > 0.$$
Reconstruction algorithm: variant of TVR-DART

We mostly follow [Zhuge, Palenstijn & Batenburg 2016] in the implementation of TVR-DART.

However, we make one bigger modification. In this application it makes a huge difference to restrict the degrees of freedom in the domain occupied by the pipe walls.
Traditional reconstruction by tomosynthesis

Simulated phantom:

Tomosynthesis:
TVR-DART with domain restriction

Simulated phantom:

TVR-DART:
Reconstructions from measured data

Tomosynthesis

TVR-DART

[Niemi, Salosensaari, Meaney & S, submitted manuscript]
Thank you for your attention!