Classifying Stroke Using Electrical Impedance Tomography

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Outline

**Electrical impedance tomography (EIT)**

Complex geometric optics (CGO) solutions, D-bar method

Application of EIT to stroke

Virtual Hybrid Edge Detection (VHED)

Combining machine learning with VHED
Electrical impedance tomography is useful for medical chest imaging

Lungs filled with air are resistive, shown as blue.

Blood in the heart appears red, as it is more conductive than air.
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos \theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 2\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 3\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 4\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 5\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns. Saline and agar phantom

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern $\cos 6\theta$

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 7\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 8\theta$

Measure the resulting voltages at all 32 electrodes

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 9\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 10\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern $\cos 11\theta$

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 12\theta$

Measure the resulting voltages at all 32 electrodes

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 13\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 14\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern \( \cos 15\theta \)

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 16\theta$

Measure the resulting voltages at all 32 electrodes

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute
Here is a reconstruction of the conductivity, computed using a nonlinear Fourier transform.

Saline and agar phantom

D-bar reconstruction

[Isaacson, Mueller, Newell & S 2004]
[Montoya 2012]

Cut-off frequency $R = 4$
The inverse conductivity problem introduced by Alberto Calderón is a mathematical model for EIT.

Let $\Omega \subset \mathbb{R}^2$ be the unit disc and let conductivity $\sigma : \Omega \to \mathbb{R}$ satisfy

$$0 < M^{-1} \leq \sigma(z) \leq M.$$

Applying voltage $f$ at the boundary $\partial \Omega$ leads to the elliptic PDE

$$\begin{cases}
\nabla \cdot \sigma \nabla u = 0 \text{ in } \Omega, \\
u|_{\partial \Omega} = f.
\end{cases}$$

The Dirichlet-to-Neumann map is a model for boundary measurements

$$\Lambda_\sigma : f \mapsto \sigma \frac{\partial u}{\partial n}|_{\partial \Omega}.$$

Calderón’s problem is to recover $\sigma$ from the knowledge of $\Lambda_\sigma$.

This is a nonlinear task and an ill-posed inverse problem.
Ill-posed inverse problems are defined as opposites of well-posed direct problems.

Hadamard (1903): a problem is well-posed if the following conditions hold.

1. A solution exists,
2. The solution is unique,
3. The solution depends continuously on the input.

Well-posed linear direct EIT problem:
Input $\sigma$, find infinite-precision data $\Lambda_{\sigma}$.

Ill-posed nonlinear inverse EIT problem:
Input noisy data $\Lambda^{\delta}_{\sigma}$, reconstruct $\sigma$. 
We illustrate the ill-posedness of EIT using a simulated example

\[ \sigma_1 \]
\[ \sigma_2 \]
We apply the voltage distribution $f(\theta) = \cos \theta$ at the boundary of the two different phantoms.
The measurement is the distribution of current through the boundary.
The current data are very similar, although the conductivities are quite different

\[ \sigma_1 \frac{\partial u_1}{\partial \vec{n}} \quad \sigma_2 \frac{\partial u_2}{\partial \vec{n}} \]
Let us apply the more oscillatory distribution $f(\theta) = \cos 2\theta$ of voltage at the boundary.
The measurement is again the distribution of current through the boundary

\[ \sigma_1 \frac{\partial u_1}{\partial \vec{n}} \]

\[ \sigma_2 \frac{\partial u_2}{\partial \vec{n}} \]
The current distribution measurements are almost the same

\[ \sigma_1 \frac{\partial u_1}{\partial \vec{n}} \quad \sigma_2 \frac{\partial u_2}{\partial \vec{n}} \]
EIT is an ill-posed problem: big differences in conductivity cause only small effect in data

$\sigma_1$

$\sigma_2$

$\cos \theta$

$\cos 2\theta$

$\cos 3\theta$

$\cos 4\theta$

$\cos 5\theta$

$\cos 6\theta$
Ghosts, or invisible structures, when using point electrodes in electrical impedance tomography

[Chesnel, Hyvönen & Staboulis 2014]
Here are the D-bar reconstructions from simulated EIT data using frequency cutoff $R = 4$.
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This part is a joint work with

David Isaacson, Rensselaer Polytechnic Institute, USA

Kim Knudsen, Technical University of Denmark

Matti Lassas, University of Helsinki, Finland

Jon Newell, Rensselaer Polytechnic Institute, USA

Jennifer Mueller, Colorado State University, USA
There exists a nonlinear Fourier transform adapted to electrical impedance tomography.
The nonlinear Fourier transform can be recovered from infinite-precision EIT measurements

\[ \Lambda_\sigma \]  

BIE

Ideal measurement

Nonlinear IFFT

[Nachman 1996]
Measurement noise prevents the recovery of the nonlinear Fourier transform at high frequencies.
We truncate away the bad part in the transform; this is a nonlinear low-pass filter.
The D-bar method is a regularization strategy for reconstructing the full conductivity distribution.

[S, Mueller & Isaacson 2000]
[Knudsen, Lassas, Mueller & S 2009]
This is a brief history of the two-dimensional regularized D-bar method for EIT

1966 Faddeev: Complex geometric optics (CGO) solutions

1987 Sylvester and Uhlmann: CGO solutions for inverse boundary-value problems; uniqueness for 3D EIT with smooth conductivities and infinite-precision data

1988 R. G. Novikov: Core ideas of the D-bar method

1988 Nachman: D-bar method for 3D EIT

1996 Nachman: Uniqueness and reconstruction for 2D EIT with $C^2$ conductivities and infinite-precision data

2000 S, Mueller and Isaacson: Numerical implementation of Nachman’s proof using a Born approximation

2006 Isaacson, Mueller, Newell and S: Application of the D-bar method to EIT data measured from a human subject

2009 Knudsen, Lassas, Mueller and S: Regularization proof
Regularized reconstructions from simulated data with noise amplitude $\|\varepsilon\| = \|\Lambda^\varepsilon - \Lambda^\sigma\|_Y$

$\|\varepsilon\| \approx 10^{-6}$

$\|\varepsilon\| \approx 10^{-5}$

$\|\varepsilon\| \approx 10^{-4}$

$\|\varepsilon\| \approx 10^{-3}$

$\|\varepsilon\| \approx 10^{-2}$

The percentages are the relative square norm errors in the reconstructions.
D-bar images can be sharpened by Deep Learning

[Hamilton & Hauptmann 2017]
Medical application of EIT and the D-bar method: quantifying air-trapping in cystic fibrosis patients

All results on this slide are from Jennifer Mueller’s group at Colorado State University.

Images: ventilation-perfusion index maps, computed from three subjects at Children’s Hospital Colorado using EIT and the D-bar method.

Dark blue regions are well-perfused but poorly ventilated.

Radiologist’s report for Subject B: extensive regions of air trapping, regional to the lung areas affected by plugging, approximately 50% of both lungs.

Healthy control
Average index 0.46

CF Subject A
Average index 0.34

CF Subject B
Average index 0.10
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Motivation of this study: imaging stroke with EIT

Ischemic stroke: low conductivity.
CT image from Jansen 2008

Hemorrhagic stroke: high conductivity.
CT image from Nakano et al. 2001

Same symptoms in both cases!

Difficulties: resistive skull layer and unknown background. However, see
- Romsauerova, McEwan, Horesh, Yerworth, Bayford & Holder 2006
- Shi, You, Xu, Wang, Fu, Liu & Dong 2009
- Bayford & Tizzard 2012
- Malone, Jehl, Arridge, Betcke & Holder 2014
- Boverman, Kao, Wang, Ashe, Davenport & Amm 2016
Brain EIT imaging is based on covering the head partly by electrodes

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Ω cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cortex</td>
<td>229</td>
</tr>
<tr>
<td>White matter</td>
<td>344</td>
</tr>
<tr>
<td>Blood</td>
<td>125</td>
</tr>
<tr>
<td>CS fluid</td>
<td>69</td>
</tr>
<tr>
<td>Scalp</td>
<td>490</td>
</tr>
<tr>
<td>Skull</td>
<td>6500</td>
</tr>
</tbody>
</table>

The current activity was initiated by Alex Ross from GE. He is a former student of David Isaacson.
The idea would be to equip every ambulance with an EIT device for classifying strokes

In David Holder’s lab at UCL
Another important application of stroke-EIT is monitoring a patient in an intensive care unit.
We have a collaboration network in place for the stroke-EIT project

Project funded for 2017–2020
- Jari Hyttinen & Antti Paldanius (U Tampere)
- Ville Kolehmainen, Asko Hänninen & Jussi Toivanen (U Eastern Finland)
- S, Matti Lassas, Minh Mach & Rashmi Murthy (U Helsinki)

Finnish collaboration:
Stefan Björkman (U Helsinki)
Valentina Candiani (Aalto U)
Antti Hannukainen (Aalto U)
Nuuetti Hyvönen (Aalto U)

International collaboration:
Juan Pablo Agnelli (U Córdoba)
Melody Alsaker (Gonzaga U)
Aynur Çöl (Sinop U)
Sarah Hamilton (Marquette U)
Andreas Hauptmann (UCL)
Kim Knudsen (DTU)
Jennifer Mueller (CSU), Toshiaki Yachimura (Tohoku)
We can test our algorithms with realistic head phantoms.

Ville Kolehmainen, Asko Hänninen, Tuomo Savolainen, Jussi Toivanen
University of Eastern Finland
We consider three simulated 2D stroke phantoms: here healthy brain
We consider three simulated 2D stroke phantoms: here ischemic stroke
We consider three simulated 2D stroke phantoms: here hemorrhagic stroke
New result: inverse scattering methods can transform EIT into “X-ray tomography”

Video:

https://www.youtube.com/watch?v=37yOcfBfRJk

[Greenleaf, Lassas, Santacesaria, S and Uhlmann 2018]
New result: inverse scattering methods can transform EIT into “X-ray tomography”

[Greenleaf, Lassas, Santacesaria, S and Uhlmann 2018]
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Allan Greenleaf, University of Rochester, NY, USA

Matti Lassas, University of Helsinki, Finland

Matteo Santacesaria, University of Helsinki, Finland

Gunther Uhlmann, University of Washington, USA
Let us analyse Complex Geometric Optics solutions using complex two-vectors

Consider complex two-vectors $\eta \in \mathbb{C}^2$ of the form

$$ \eta = (\theta_1 + i\theta_2, -\theta_2 + i\theta_1), $$

where $\theta = \theta_1 + i\theta_2 \in \mathbb{C}$ is a unitary complex number: $|\theta| = 1$.

Denote a planar point by $x = (x_1, x_2) \in \mathbb{R}^2$. Because $\eta \cdot \eta = 0$ we see that $e^{i\tau \eta \cdot x} = 0$ is harmonic in $x$.

We next analyse solutions of the conductivity equation $\nabla \cdot \sigma \nabla u = 0$ of the form

$$ u(x) = e^{i\tau \eta \cdot x} w(x, \tau). $$

(They are connected to the previously discussed CGO solutions via $k = \tau \theta$ and $z = x_1 + i x_2$, since then $e^{ikz} = e^{i\tau \theta z} = e^{i\tau \eta \cdot x}$.)
Since \( u(x) = e^{i\tau \eta \cdot x} w(x, \tau) \) satisfies the conductivity equation,

\[
0 = \frac{1}{\sigma(x)} \nabla \cdot (\sigma(x) \nabla u(x))
\]

\[
= (\Delta + \frac{1}{\sigma}(\nabla \sigma) \cdot \nabla)(e^{i\tau \eta \cdot x} w(x, \tau))
\]

\[
= \left( \Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot (\nabla + i\tau \eta) w(x, \tau) \right) e^{i\tau \eta \cdot x}.
\]

Hence, we have

\[
\Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot (\nabla + i\tau \eta) w(x, \tau) = 0.
\]
New trick: apply one-dimensional Fourier transform to the complex spectral parameter

Let \( \hat{w}(x, t) \) be the Fourier transform of \( w(x, \tau) \) in the \( \tau \) variable:

\[
\hat{w}(x, t) = \mathcal{F}w(x, t) = \int_{-\infty}^{\infty} e^{-it\tau} w(x, \tau) \, d\tau.
\]

We call \( t \) the *pseudo-time* corresponding to complex frequency \( \tau \). Then equation

\[
\Delta w(x, \tau) + 2i\tau \eta \cdot \nabla w(x, \tau) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot \left( \nabla + i\tau \eta \right) w(x, \tau) = 0
\]

yields

\[
\Delta \hat{w}(x, t) + 2\eta \frac{\partial}{\partial t} \cdot \nabla \hat{w}(x, t) + \left( \frac{1}{\sigma} \nabla \sigma \right) \cdot \left( \nabla + \eta \frac{\partial}{\partial t} \right) \hat{w}(x, t) = 0.
\]
Complex principal type operator leads to singularities propagating along leaves

Denote $\eta = \eta_R + i \eta_I$. The principal part of the equation

$$\Delta \hat{w}(x, t) + 2\eta \frac{\partial}{\partial t} \cdot \nabla \hat{w}(x, t) + \frac{1}{\sigma} (\nabla \sigma) \cdot (\nabla + \eta \frac{\partial}{\partial t}) \hat{w}(x, t) = 0$$

is given by the complex principal type operator

$$\Delta + 2\eta \frac{\partial}{\partial t} \cdot \nabla = \left( \Delta + 2\eta_R \frac{\partial}{\partial t} \cdot \nabla \right) + i \left( 2\eta_I \frac{\partial}{\partial t} \cdot \nabla \right),$$

in the sense of Duistermaat and Hörmander (1972).

For a real principal type operator the characteristic singularities propagate along one-dimensional rays. For instance, for the wave equation the light-like singularities propagate along light rays.

For a complex principal type operator the characteristic singularities propagate along two-dimensional surfaces called *leaves*. 
Let us consider a rotationally symmetric conductivity with jump along the circle $|z| = 0.2$. 

$\begin{align*}
z &= 1 \\
k &= \tau e^{i\varphi} \\
\varphi &= 0
\end{align*}$
We use propagation and reflection of singularities along leaves for detecting inclusions.

Here the magenta plane wave hits the blue surface, producing light blue reflected waves.
Here are the two lowest-order odd terms in the scattering series, with subtraction

\[ [T_1^+-T_1^-]\mu \quad [T_3^+-T_3^-]\mu \]
Recovery by complex “filtered back-projection”

Theorem. (Greenleaf, Lassas, Santacesaria, S and Uhlmann 2018)
Define averaged operators $T_{j}^{\pm}$ for $j = 1, 2, 3, \ldots$ by the complex contour integral:

$$T_{j}^{\pm} \mu(t, e^{i\varphi}) = \frac{1}{2\pi i} \int_{\partial \Omega} \hat{\omega}_{j}^{\pm}(z, t, e^{i\varphi})dz,$$

Then we have a filtered back-projection formula

$$(-\Delta)^{-1/2}(T_{1}^{\pm})^{*} T_{1}^{\pm} \mu = \mu.$$
Simple example of tomographic imaging with a double-disc target

https://youtu.be/5DUGTXd26nA
We can back-project the measured data into the image, integrating over all directions

https://youtu.be/5DUGTXd26nA
Final FBP reconstruction involves filtering on top of the back-projection

Multiplication by $|\xi|$ (Calderón’s operator)
Conductivity Filtered back-projection “Λ-tomography”
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**Aynur Çöl**, Sinop University, Turkey

**Allan Greenleaf**, University of Rochester, NY, USA

**Matti Lassas**, University of Helsinki, Finland

**Minh Mach**, University of Helsinki, Finland

**Rashmi Murthy**, University of Helsinki, Finland

**Matteo Santacesaria**, University of Helsinki, Finland

**Gunther Uhlmann**, University of Washington, USA

**Toshiaki Yachimura**, Tohoku University, Japan
Given unrealistic-precision EIT measurements on full boundary we can classify the stroke easily.

\[ \int_{-60}^{60} \Phi_{60}(\tau) e^{-it\tau} \omega^\pm(z, \tau, e^{i\varphi}) \, d\tau \]
Remember the noise-induced stable and unstable parts of the nonlinear frequency domain
Practical EIT measurements blur the information due to heavily windowed Fourier transform

\[ \int_{-4}^{4} \Phi_4(\tau) e^{-it\tau} \omega^{\pm}(z, \tau, e^{i\varphi}) \, d\tau \]
Perhaps machine learning will help us?
We simulate a set of 5000 conductivities with ischemic/hemorrhagic stroke on right hemisphere
We simulate a set of 5000 conductivities with ischemic/hemorrhagic stroke on right hemisphere
We simulate a set of 5000 conductivities with ischemic/hemorrhagic stroke on right hemisphere.
The conductivities have random parameters

- **Boundary**: \( \sigma(x) = 1 \)
- **Skull**: \( \sigma(x) \in [0.45, 0.55] \)
- **Stroke**: \( \sigma(x) \in [3, 4] \)
- **Background**: \( \sigma(x) \in [0.7, 0.8] \)
Preliminary results on using VHED as a nonlinear feature for machine learning

We trained each Fully Connected Neural Network (FCNN) using the 5000 disc inclusions simulated in Experiment 1 and then we tested each network using 3500 samples corresponding to disc inclusions with the properties described in Experiment 2.

The output of the network is a number between 0 and 1: zero for ischemic stroke and 1 for hemorrhage.

<table>
<thead>
<tr>
<th></th>
<th>Margin 0.5</th>
<th>Margin 0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND matrix</td>
<td>0.982</td>
<td>0.979</td>
</tr>
<tr>
<td>DN matrix</td>
<td>0.985</td>
<td>0.970</td>
</tr>
<tr>
<td>VHED</td>
<td><strong>0.999</strong></td>
<td><strong>0.997</strong></td>
</tr>
</tbody>
</table>

[Agnelli, Çöl, Murthy and Siltanen, unpublished results]
Links to open computational resources

Open EIT datasets:

- Finnish Inverse Problems Society (FIPS) dataset page

Reconstruction algorithms: FIPS Computational Blog

- The D-bar Method for EIT—Simulated Data
- The D-bar Method for EIT—Experimental Data

For these slides see: http://www.siltanen-research.net/talks.html
Thank you for your attention!