The mathematics of X-ray tomography

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EDUFI Winter School, Tvärminne, March 7, 2018
This my industrial-academic background

2009: Professor, University of Helsinki, Finland

2006: Professor, Tampere University of Technology, Finland

2005: R&D scientist at Palodex Group

2004: R&D scientist at GE Healthcare

2002: Postdoc at Gunma University, Japan

2000: R&D scientist at Instrumentarium Imaging

1999: PhD, Helsinki University of Technology, Finland
Samun tiedekanava
Lotus root tomography

YouTube search: “lotus tomography”

www.youtube.com/watch?v=eWwD_EZuzBI&t=7s

Video: thanks to Tatiana Bubba, Andreas Hauptmann and Juho Rimpeläinen
Outline

X-ray imaging

Tomographic imaging with sparse data

Industrial case study: low-dose 3D dental X-ray imaging

What is back-projection?
Wilhelm Conrad Röntgen invented X-rays and was awarded the first Nobel Prize in Physics in 1901.
We can get 3D information by imaging along different directions
Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography.

Hounsfield (top) and Cormack received Nobel prizes in 1979.
Diagnosing stroke with X-ray tomography

Ischemic stroke

CT image from Jansen 2008

Hemorrhagic stroke

CT image from Nakano et al. 2001
X-ray intensity attenuates inside matter, here shown with a homogeneous block

https://www.youtube.com/watch?v=IfXo2S1xXCQ
Here is a more complicated example: a 2D slice through a human head

Andrew Ciscel, Wikimedia commons
Now the attenuation process is more complicated because there are different tissues

https://youtu.be/lvUAOeS1sv8
After calibration we are observing how much attenuating matter the X-ray encounters in total.

https://youtu.be/RFArLtWEfsQ
This sweeping movement is the data collection mode of first-generation CT scanners

https://youtu.be/JHUz5oyeZb0
Data is collected by rotating the system around the patient

https://youtu.be/newxZbw7YAs
Rotating around the object allows us to form the so-called *sinogram*

https://youtu.be/q7Rt_OY_7tU
Modern CT scanners look like this
Modern scanners rotate at high speed

https://commons.wikimedia.org/wiki/File:CT-Rotation.ogv
This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAqI
This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58lO1FKw
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What is back-projection?
We collected X-ray projection data of a walnut from 1200 directions.

Laboratory and data collection by Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää.
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

FBP with comprehensive data (1200 projections)

FBP with sparse data (20 projections)
Sparse-data reconstruction of the walnut using non-negative total variation regularization

\[
\text{arg min}_{f \in \mathbb{R}^n_+} \left\{ \| Af - m \|_2^2 + \alpha \| \nabla f \|_1 \right\}
\]
**TV tomography:** \( \arg \min_{f \in \mathbb{R}^n} \{ \| Af - m \|_2^2 + \alpha \| \nabla f \|_1 \} \)

1992 Rudin, Osher & Fatemi: denoise images by taking \( A = I \)

1998 Delaney & Bresler

2001 Persson, Bone & Elmqvist

2003 Kolehmainen, S, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä & Somersalo (first TV work with measured X-ray data)

2006 Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke

2006 Sidky, Kao & Pan

2008 Liao & Sapiro

2008 Sidky & Pan

2008 Herman & Davidi

2009 Tang, Nett & Chen

2009 Duan, Zhang, Xing, Chen & Cheng

2010 Bian, Han, Sidky, Cao, Lu, Zhou & Pan

2011 Jensen, Jørgensen, Hansen & Jensen

2011 Tian, Jia, Yuan, Pan & Jiang

2012–present: dozens of articles indicated by Google Scholar
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What is back-projection?
The VT device was developed in 2001–2012 by

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
Application: dental implant planning, where a missing tooth is replaced with an implant
Dental X-ray imaging 100 years ago
It is tedious to interpret a mosaic of overlapping intraoral X-ray images.
Panoramic dental imaging shows all the teeth simultaneously.

Panoramic imaging was invented by Yrjö Veli Paatero in the 1950’s.
This is the classical imaging procedure of the panoramic X-ray device

https://www.youtube.com/watch?v=QFTXegPxC4U
The resulting image shows a sharp layer positioned inside the dental arc.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics. A panoramic dental image offers a general overview showing all teeth and other structures simultaneously. Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
We reprogram the panoramic X-ray device so that it collects projection data by scanning

https://www.youtube.com/watch?v=motthjiP8ZQ
We reprogram the panoramic X-ray device so that it collects projection data by scanning

Number of projection images: 11

Angle of view: 40 degrees

Image size: $1000 \times 1000$ pixels

The unknown vector $f$ has 7,000,000 elements.
Standard Cone Beam CT reconstruction delivers 100 times more radiation than VT imaging

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke 2006
Kolehmainen, Lassas & S 2008
Cederlund, Kalke & Welander 2009
Hyvönen, Kalke, Lassas, Setälä & S 2010
U.S. patent 7269241, thousands of VT units in use
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What is back-projection?
Simple example of tomographic imaging with a double-disc target

https://youtu.be/5DUGTXd26nA
Tomografian käänteinen ongelma on löytää kuvattava kohde, kun vain röntgenkuvat tunnetaan

https://youtu.be/YhClb0MaB70
Since we know the projection directions, we can back-project the data into the image.
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Back-projection becomes more useful by summing up the images
Summing all the back-projections results in a blurred reconstruction.
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Summing all the back-projections results in a blurred reconstruction
Here we use more directions, so the reconstruction quality is higher

https://youtu.be/5DUGTXd26nA
Final reconstruction involves filtering on top of the back-projection.
Thank you for your attention!