Dynamic X-ray tomography with level set regularization

Samuli Siltanen, University of Helsinki, Finland

International Congress on Industrial and Applied Mathematics
Minisymposium Computational Inverse Problems
Beijing, China, August 14, 2015
http://www.siltanen-research.net
Finnish Centre of Excellence in Inverse Problems Research

http://wiki.helsinki.fi/display/inverse/Home
This is a joint work with

Keijo Hämäläinen, University of Helsinki, Finland

Lauri Harhanen, Technical University of Denmark

Aki Kallonen, University of Helsinki, Finland

Ville Kolehmainen, University of Eastern Finland

Matti Lassas, University of Helsinki, Finland

Esa Niemi, University of Helsinki, Finland
Outline

Motivation: multi-source dynamic tomography

Spacetime regularization with level sets

Reconstructions with (2+1)D simulated data

Reconstructions with measured data
We study a tomographic imaging modality based on multiple source-detector pairs

Consider placing many X-ray sources and detectors in fixed positions in 3D.

Digital flat-panel X-ray detectors are available with 400Hz frame-rate, giving projection videos.

Reconstructing the 3D structure at all times leads to 4D tomography.

Applications include cardiac imaging, angiography, biotechnology research, veterinary medicine, nondestructive testing.
One potential benefit of this imaging modality is three-dimensional angiography.

This is regular two-dimensional angiography.

Video by Dr. Magda Bayoumi, downloaded from Dailymotion
Very brief overview of multi-source tomographic studies, all based on FBP-type algorithms

1980 Berninger & Redington: Multiple purpose high speed tomographic x-ray scanner (patent)

1983 Robb, Hoffman, Sinak, Harris & Ritman: High-speed three-dimensional x-ray computed tomography: The dynamic spatial reconstructor

1993 Stiel, Stiel, Klotz & Nienaber: Digital flashing tomosynthesis: a promising technique for angiocardiographic screening


Static multi-source arrangements have received very little attention in the literature. Filtered back-projection type methods are not well-suited for the resulting sparse datasets.
Reconstruction methods for dynamic tomography

1997 Baroudi & Somersalo: Gas temperature mapping using impedance tomography

2002 Lu & Mackie: Tomographic motion detection and correction directly in sinogram space

2003 Bonnet et al.: Dynamic X-Ray Computed Tomography

2004 Roux et al.: Exact reconstruction in 2D dynamic CT: compensation of time-dependent affine deformations

2006 Kindermann & Leitão: On regularization methods for inverse problems of dynamic type

2010 Katsevich: An accurate approximate algorithm for motion compensation in two-dimensional tomography

2014 Hahn: Reconstruction of dynamic objects with affine deformations in computerized tomography

2015 Hahn: Dynamic linear inverse problems with moderate movements of the object: Ill-posedness and regularization
Outline

Motivation: multi-source dynamic tomography

Spacetime regularization with level sets

Reconstructions with (2+1)D simulated data

Reconstructions with measured data
The level set method can be used to model mud
The level set method [Osher, Sethian] parametrizes curves and surfaces in a flexible way.
Selected papers on level set methodology

1988 Osher & Sethian: Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations

1998 Harabetian & Osher: Regularization of ill-posed problems via the level set approach

2005 Burger & Osher: A survey on level set methods for inverse problems and optimal design

2006 Dorn & Lesselier: Level set methods for inverse scattering

2007 Ramlau & Ring: A Mumford-Shah level-set approach for the inversion and segmentation of X-ray tomography data

2008 Kolehmainen, Lassas & Siltanen: Limited data X-ray tomography using nonlinear evolution equations

2011 Klann, Ramlau, Ring: A Mumford-Shah level-set approach for the inversion and segmentation of SPECT/CT data

2013 Niemi, Lassas & S: Dynamic X-ray tomography with multiple sources
A generalization of the classical level set method was introduced in [Kolehmainen, Lassas & S 2008]

We model the X-ray attenuation function as $g(\Phi(x, y))$, where

$$g(\tau) = \begin{cases} 
\tau, & \text{if } \tau \geq 0 \\
0, & \text{if } \tau < 0.
\end{cases}$$

The smooth level set function $\Phi(x, y) := \lim_{s \to \infty} \phi(x, y, s)$ is the large-time limit of the solution of the evolution equation

$$\begin{align*}
\phi_s &= -A^*(A(g(\phi)) - m) + \beta \Delta \phi, \\
(\nu \cdot \nabla - r)\phi|_{\partial \Omega} &= 0,
\end{align*}$$

with a suitable initial condition. Here $\beta > 0$, $r \geq 0$, $A^*$ denotes the transpose of $A$, and $\Delta \phi = \phi_{xx} + \phi_{yy}$.
The generalized level set method works nicely for stationary limited-angle 2D tomography.

Images from [Kolehmainen, Lassas & S 2008]. However, see also [Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
We deal with the dynamic case by considering the moving target in spacetime
We write a higher-order level set method in (2+1)D spacetime for the dynamic case.

**The 2D static case:**

We model the X-ray attenuation as $g(\phi(x, y))$. The level set function $\phi$ belongs to $H^1(\Omega)$ and is defined as the minimizer of

$$\|A g(\phi) - m\|_{L^2}^2 + \alpha \|\nabla \phi\|_{L^2}^2,$$

where $\nabla \phi = [\phi_x, \phi_y]^T$.

[Kolehmainen, Lassas & S 2008]

**The (2+1)D dynamic case:**

We model the X-ray attenuation as $g(\Phi(x, y, t))$. The level set function $\phi$ belongs to $H^2(\Omega \times [0, T])$ and is defined as the minimizer of

$$\|A g(\phi) - m\|_{L^2}^2 + \alpha \|\nabla \phi\|_{L^2}^2 + \alpha(\|\partial^2_x \phi\|_{L^2}^2 + \|\partial^2_y \phi\|_{L^2}^2 + \|\partial^2_t \phi\|_{L^2}^2),$$

where $\nabla \phi = [\phi_x, \phi_y, \phi_t]^T$.

[Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
There exists at least one minimizer for our higher-order functional

**Theorem:** Let $\mathcal{A}$ be an operator modeling 2D Radon transforms measured at several times. Then the functional

$$F_n(\phi) := \frac{1}{2} \| \mathcal{A}g(\phi) - m \|_2^2 + \frac{\alpha}{2} \sum_{1 \leq |\beta| \leq n} \| D^\beta \phi \|_2^2,$$

has a **global minimizer**. The minimizer is unique for $n = 1$. 
Numerical minimization in the case $n = 2$

We smooth out the nondifferentiability of the objective functional by replacing $g : \mathbb{R} \to \mathbb{R}$ by the differentiable approximation

$$g_\delta(\tau) = \begin{cases} \sqrt{\tau^2 + \delta^2} - \delta, & \text{if } \tau > 0, \\ 0, & \text{if } \tau \leq 0, \end{cases}$$

where $\delta > 0$ is small.

Now we can use a gradient-based optimization method for computing the minimizer of

$$\|Ag_\delta(\phi) - m\|_{L^2}^2 + \alpha \|\nabla \phi\|_{L^2}^2 + \alpha (\|\partial_x^2 \phi\|_{L^2}^2 + \|\partial_y^2 \phi\|_{L^2}^2 + \|\partial_t^2 \phi\|_{L^2}^2).$$
Outline

Motivation: multi-source dynamic tomography

Spacetime regularization with level sets

Reconstructions with $(2+1)D$ simulated data

Reconstructions with measured data
Two simulated examples, based on only seven (7) projection directions:

Imaging geometry:

Spacetime phantoms:
<table>
<thead>
<tr>
<th>Original</th>
<th>Level set ( n = 1 )</th>
<th>Level set ( n = 2 )</th>
<th>Tikhonov 2D, ( \geq 0 )</th>
<th>TV 2D, ( \geq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Original Image]</td>
<td>![Level Set Image 1]</td>
<td>![Level Set Image 2]</td>
<td>![Tikhonov Image]</td>
<td>![TV Image]</td>
</tr>
<tr>
<td>![Original Image]</td>
<td>![Level Set Image 1]</td>
<td>![Level Set Image 2]</td>
<td>![Tikhonov Image]</td>
<td>![TV Image]</td>
</tr>
<tr>
<td>![Original Image]</td>
<td>![Level Set Image 1]</td>
<td>![Level Set Image 2]</td>
<td>![Tikhonov Image]</td>
<td>![TV Image]</td>
</tr>
<tr>
<td>![Original Image]</td>
<td>![Level Set Image 1]</td>
<td>![Level Set Image 2]</td>
<td>![Tikhonov Image]</td>
<td>![TV Image]</td>
</tr>
</tbody>
</table>

Note: The images show the original and processed results using different methods.
Level set reconstruction in spacetime

Original phantom
Original

<table>
<thead>
<tr>
<th>Original</th>
<th>Level set $n = 1$</th>
<th>Level set $n = 2$</th>
<th>Tikhonov 2D, $\geq 0$</th>
<th>TV 2D, $\geq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
</tr>
</tbody>
</table>

[Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
Level set reconstruction in spacetime

Original phantom
The level set functions with $n = 1$ and $n = 2$ for the middle slice in the previous example.
Outline

Motivation: multi-source dynamic tomography

Spacetime regularization with level sets

Reconstructions with (2+1)D simulated data

Reconstructions with measured data
X-ray sources and detectors are expensive, and currently we have only one of each.

How to use one X-ray source and one detector to create a multi-source type dataset?
Consider a simple multi-source measurement:

\[ t = 1 \]
Consider a simple multi-source measurement:

\[ t = 2 \]
Consider a simple multi-source measurement:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 1 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 1 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 1 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 2 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 2 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 2 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
Level set ($n = 2$) reconstruction from 10 projections

$t = 1$

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections
FBP reconstruction from 120 projections

\[ t = 2 \]

Tomographic data: Keijo Hämäläinen
Aki Kallonen
Reconstruction: Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 3 \]

**Tomographic data:**
Keijo Hämäläinen
Aki Kallonen

**Reconstruction:**
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 4 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
Level set ($n = 2$) reconstruction from 10 projections

$\mathbf{t} = 5$

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections
FBP reconstruction from 120 projections

\[ t = 6 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
Level set \((n = 2)\) reconstruction from 10 projections

\[ t = 7 \]

**Tomographic data:**
Keijo Hämäläinen
Aki Kallonen

**Reconstruction:**
Esa Niemi

FBP reconstruction from 120 projections
Level set \((n = 2)\) reconstruction from 10 projections

\[ t = 8 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen

Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections
Level set ($n = 2$) reconstruction from 10 projections

FBP reconstruction from 120 projections

$t = 9$

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set ($n = 2$) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 10 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
This is a movie showing the recovered level set in (2+1) dimensional spacetime

Computation and visualization by Esa Niemi
The next step: 4D imaging of moving objects

Data and video: thanks to Alexander Meaney and Topias Rusanen
Lego robot under imaging
Lego robot under imaging
This is simply a SIRT reconstruction from 360 views using the ASTRA toolbox

Computation and visualization by Topias Rusanen
Part I: Linear Inverse Problems
1 Introduction
2 Naïve reconstructions and inverse crimes
3 Ill-Posedness in Inverse Problems
4 Truncated singular value decomposition
5 Tikhonov regularization
6 Total variation regularization
7 Besov space regularization using wavelets
8 Discretization-invariance
9 Practical X-ray tomography with limited data
10 Projects

Part II: Nonlinear Inverse Problems
11 Nonlinear inversion
12 Electrical impedance tomography
13 Simulation of noisy EIT data
14 Complex geometrical optics solutions
15 A regularized D-bar method for direct EIT
16 Other direct solution methods for EIT
17 Projects

All Matlab codes freely available on a website!
We collected X-ray projection data of a walnut from 1200 directions.

Laboratory and data collection by Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää.
Sparse-data reconstruction of the walnut using non-negative total variation regularization

Filtered back-projection

Constrained TV regularization

$$\arg \min_{f \in \mathbb{R}^n_+} \{ ||Af - m||_2^2 + \alpha ||\nabla f||_1 \}$$
This is my new X-ray laboratory at University of Helsinki