Gray-box machine learning for electrical impedance tomography

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Joint work with Takanori Ide (Aisin AW, Riken)
Outline

Electrical impedance tomography (EIT)

The enclosure method: theory

The enclosure method: least-squares computation

The enclosure method: deep learning
This section concentrates on applications of EIT to chest imaging

Medical applications: monitoring cardiac activity, lung function, and pulmonary perfusion. Also, electrocardiography (ECG) can be enhanced using knowledge about conductivity distribution.
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern \( \cos \theta \)

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 2\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 3\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 4\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 5\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern $\cos 6\theta$

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 7\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 8\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 9\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Apply current pattern $\cos 10\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns

Saline and agar phantom

Applies current pattern $\cos 11\theta$

Measure the resulting voltages at all 32 electrodes

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern $\cos 12\theta$

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

[Image of a saline and agar phantom]

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Apply current pattern $\cos 13\theta$

[Diagram showing current patterns]

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 14\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern \( \cos 15\theta \)

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes.
Note that EIT data collection involves applying several current patterns.

Saline and agar phantom

Apply current pattern $\cos 16\theta$

Phantom and data: Jon Newell, Rensselaer Polytechnic Institute

Measure the resulting voltages at all 32 electrodes
Here is a reconstruction of the conductivity, computed using a nonlinear Fourier transform.

Saline and agar phantom

D-bar reconstruction

[Isaacson, Mueller, Newell & S 2004]
[Montoya 2012]

Cut-off frequency $R = 4$
Medical application of EIT and the D-bar method: quantifying air-trapping in cystic fibrosis patients

All results on this slide are from Jennifer Mueller’s group at Colorado State University.

Images: ventilation-perfusion index maps, computed from three subjects at Children’s Hospital Colorado using EIT and the D-bar method.

Dark blue regions are well-perfused but poorly ventilated.

Radiologist’s report for Subject B: extensive regions of air trapping, regional to the lung areas affected by plugging, approximately 50% of both lungs.

Healthy control
Average index 0.46

CF Subject A
Average index 0.34

CF Subject B
Average index 0.10
The idea would be to equip every ambulance with an EIT device for classifying strokes

In David Holder’s lab at UCL
Another important application of stroke-EIT is monitoring a patient in an intensive care unit.
We have a collaboration network in place for the stroke-EIT project

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Outline

Electrical impedance tomography (EIT)

The enclosure method: theory

The enclosure method: least-squares computation

The enclosure method: deep learning
The mathematical model of EIT is the inverse conductivity problem introduced by Calderón

Let \( \Omega \subset \mathbb{R}^2 \) be the unit disc and let conductivity \( \sigma : \Omega \to \mathbb{R} \) satisfy

\[
0 < M^{-1} \leq \sigma(z) \leq M.
\]

Applying voltage \( f \) at the boundary \( \partial \Omega \) leads to the elliptic PDE

\[
\begin{cases}
\nabla \cdot \sigma \nabla u = 0 \text{ in } \Omega, \\
u|_{\partial \Omega} = f.
\end{cases}
\]

Boundary measurements are modelled by the Dirichlet-to-Neumann map

\[
\Lambda_\sigma : f \mapsto \sigma \frac{\partial u}{\partial \vec{n}}|_{\partial \Omega}.
\]

Calderón’s problem is to recover \( \sigma \) from the knowledge of \( \Lambda_\sigma \). It is a nonlinear and ill-posed inverse problem.
The inclusion detection problem

Let $\Omega \subset \mathbb{R}^2$ be a bounded, simply connected $C^\infty$ domain and

$$\sigma = 1 + \chi_D h.$$ 

Here $D \subset \Omega$ and $h \in L^\infty(D)$ is such that $\sigma$ is bounded away from zero and has a jump along $\partial D$. We assume that the conductivity $\sigma$ is strictly positive.

The goal of inclusion detection is to use EIT measurements, here $\Lambda_\sigma$, to extract information about $D$.

[Ikehata 2000], [Ikehata and S 2000], [Brühl and Hanke 2000]
The support function $h_D(\omega)$

Define the support function of inclusion $D \subset \mathbb{R}^2$ with direction $\omega \in S^1$ by

$$h_D(\omega) := \sup_{x \in D} x \cdot \omega.$$ 

Here $\Omega$ is the unit disc.
The jump condition

For $\omega \in S^1$ and $\delta > 0$, define

$$D_{\omega}(\delta) = \{ x \in D \mid h_D(\omega) - \delta < x \cdot \omega \leq h_D(\omega) \}.$$

The conductivity $\sigma = 1 + \chi_D h$ satisfies the jump condition if for each $\omega \in S^1$, there exist positive constants $C_\omega$ and $\delta_\omega$ such that

$$h(x) \geq C_\omega \quad \text{for almost all } x \in D_{\omega}(\delta_\omega), \text{ or}$$

$$-h(x) \geq C_\omega \quad \text{for almost all } x \in D_{\omega}(\delta_\omega).$$
Illustration of the jump condition

\[ h_D(\omega) - \delta \]

OK.
Illustration of the jump condition

$\omega - D(\omega) - \delta$
Calderón’s exponential

Let $\omega$ be a unit vector, and denote by $\omega^\perp$ the vector $\omega$ rotated counterclockwise by angle $\pi/2$. Then $\omega \cdot \omega^\perp = 0$.

For each $\tau > 0$ and $x \in \mathbb{R}^2$ set $f_\omega(x; \tau) := e^{\tau x \cdot \omega + i \tau x \cdot \omega^\perp}$. 
Calderón’s exponential

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For each $\tau > 0$ and $x \in \mathbb{R}^2$ set $f_\omega(x; \tau) := e^{\tau x \cdot \omega + i \tau x \cdot \omega^\perp}$.

In the plot we have $\omega = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]^T$. 

\[\begin{array}{c}
\tau=1 \\
\text{Re}(f_\omega(x; \tau)) \\
\text{Im}(f_\omega(x; \tau))
\end{array}\]
Calderón’s exponential

Let $\omega$ be a unit vector, and denote by $\omega^\perp$ the vector $\omega$ rotated counterclockwise by angle $\pi/2$. Then $\omega \cdot \omega^\perp = 0$.

For each $\tau > 0$ and $x \in \mathbb{R}^2$ set $f_\omega(x; \tau) := e^{\tau x \cdot \omega + i \tau x \cdot \omega^\perp}$.

In the plot we have $\omega = \left[ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]^T$. 

![Plot](image-url)
Calderón’s exponential

Let \( \omega \) be a unit vector, and denote by \( \omega^\perp \) the vector \( \omega \) rotated counterclockwise by angle \( \pi/2 \). Then \( \omega \cdot \omega^\perp = 0 \).

For each \( \tau > 0 \) and \( x \in \mathbb{R}^2 \) set \( f_\omega(x; \tau) := e^{\tau x \cdot \omega + i \tau x \cdot \omega^\perp} \).

In the plot we have \( \omega = \left[ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right] \).
Calderón’s exponential

Let $\omega$ be a unit vector, and denote by $\omega^\perp$ the vector $\omega$ rotated counterclockwise by angle $\pi/2$. Then $\omega \cdot \omega^\perp = 0$.

For each $\tau > 0$ and $x \in \mathbb{R}^2$ set $f_\omega(x; \tau) := e^{\tau x \cdot \omega + i \tau x \cdot \omega^\perp}$.

In the plot we have $\omega = \left[ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right]^T$. 

![Plot showing real and imaginary parts of $f_\omega(x; \tau)$](image-url)

$\tau = 20$

$\text{Re}(f_\omega(x; \tau))$

$\text{Im}(f_\omega(x; \tau))$
Calderón’s exponential and Ikehata’s indicator function

Let $\omega$ be a unit vector, and denote by $\omega^\perp$ the vector $\omega$ rotated counterclockwise by angle $\pi/2$. Then $\omega \cdot \omega^\perp = 0$.

For each $\tau > 0$ and $x \in \mathbb{R}^2$ set

$$f_\omega(x; \tau) := e^{\tau x \cdot \omega} + i \tau x \cdot \omega^\perp.$$

The indicator function $I_\omega(\tau)$ is defined by

$$I_\omega(\tau) = \langle (\Lambda_\sigma - \Lambda_1)\overline{f_\omega(\cdot; \tau)}, f_\omega(\cdot; \tau) \rangle.$$

The fundamental theorem of enclosure method says that

$$h_D(\omega) = \lim_{\tau \to \infty} \frac{\log |I_\omega(\tau)|}{2\tau},$$
Here are a few examples of convex hulls calculated with 45 uniformly distributed vectors $\omega$. 
Outline

Electrical impedance tomography (EIT)

The enclosure method: theory

The enclosure method: least-squares computation

The enclosure method: deep learning
We recover the support function approximately using least-squares fitting of a line to $\frac{1}{2} \log |I_\omega(\tau)|$.
We recover the support function approximately using least-squares fitting of a line to $\frac{1}{2} \log |l_\omega(\tau)|$.
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How about a more difficult case?
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\[ \frac{1}{2} \log |l_\omega(\tau)| \]
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The enclosure method: least-squares computation

The enclosure method: deep learning
We give the indicator functions to the neural net in the form of a $21 \times 50$ image.

Horizontal axis: angles $1, 2, \ldots, 44, 45, 1, 2, 3, 4, 5$.
Vertical axis: 7 values of indicator function thrice.
We give the indicator functions to the neural net in the form of a $21 \times 50$ image.

Horizontal axis: angles 1, 2, ..., 44, 45, 1, 2, 3, 4, 5.
Vertical axis: 7 values of indicator function thrice.
After many experiments we ended up choosing this network architecture

There are six layers in our convolutional neural network:

- **Image Input**: 21x50 images with zero mean normalization
- **Convolution**: 120 filters of size 6x4, three-pixel zero padding
- **Leaky ReLU**: Leaky ReLU with scale 0.01
- **Fully Connected**: Fully connected layer with 45 outputs
- **Tanh**: Hyperbolic tangent
- **Regression Output**
Training data: 19,000 cases with round inclusions

Background: homogeneous conductivity 1 defined in the unit disc.

There are 1–4 disc-shaped inclusions, random centers.

Disc radii are random, uniformly distributed in the interval [0.05, 0.2].

The conductivity inside each inclusion is random with uniform distribution in either [1.5, 5] or in [0.2, 0.667].
Training data: 19 000 cases with round inclusions

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The conductivity inside each inclusion is random with uniform distribution in either \([1.5, 5]\) or in \([0.2, 0.67]\).
Denote the true convex hull by $C \subset \mathbb{R}^2$ and the reconstructed convex hull by $B \subset \mathbb{R}^2$. How to measure the quality of $B$ as an approximation to $C$ quantitatively?

We use the relative error

$$\frac{|C \setminus B| + |B \setminus C|}{|C|} \cdot 100\%,$$

where $| \cdot |$ denotes area.
Reconstruction examples

LS fit error 1235%  AI error 7%  True convex hull
Reconstruction examples

LS fit error 59%

AI error 32%

True convex hull
Reconstruction examples

LS fit error 110%

AI error 21%

True convex hull
Reconstruction examples

LS fit error 116%  AI error 16%  True convex hull
Reconstruction examples

LS fit error 66%

AI error 7%

True convex hull
Test data: 1 000 cases with elliptic inclusions

Background: homogeneous conductivity 1 defined in the unit disc.

Five randomly placed and oriented elliptic inclusions.

Each semi-major axis random with half-length $R$ uniformly distributed in the interval $[0.04, 0.22]$; each semi-minor axis $r$ in the interval $[0.04, R]$.

The conductivity inside each inclusion is random with uniform distribution in either $[1.3, 7]$ or in $[0.14, 0.77]$. 
Test data: 1000 cases with elliptic inclusions

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The conductivity inside each inclusion is random with uniform distribution in either $[1.3, 7]$ or in $[0.14, 0.77]$. 
Reconstruction examples: test data

LS fit error 99%

AI error 12%

True convex hull
Reconstruction examples: test data

LS fit error 71%

AI error 9%

True convex hull
Reconstruction examples: test data

LS fit error 83%  AI error 8%  True convex hull
Reconstruction examples: test data

LS fit error 89%  AI error 12%  True convex hull
Reconstruction examples: test data

LS fit error 64%  AI error 12%  True convex hull
Error histograms for least-squares approach and AI

**Least-squares fit relative errors (%)**

**Deep learning relative errors (%)**
Conclusion

When solving ill-posed inverse problems with neural networks, use a classical inversion method for extracting robust features (as opposed to learning from raw data).

Analytic understanding of the features leads to gray-box learning (instead of black-box learning).

Future work

- Add comparison to enhanced least-squares approach
- Compare the above approach to learning directly from raw data
- Use the complete electrode model
- Work with measured data
Links to open computational resources

Open EIT datasets:
- Finnish Inverse Problems Society (FIPS) dataset page

Reconstruction algorithms: FIPS Computational Blog
- The D-bar Method for EIT—Simulated Data
- The D-bar Method for EIT—Experimental Data

For these slides see: http://www.siltanen-research.net/talks.html
Thank you for your attention!