Multiresolution parameter choice method for TV —the case of sparse-data X-ray tomography

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IFIP TC 7.4 Workshop on Inverse Problems and Imaging
Akademie Wolfsburg, Mülheim an der Ruhr, Germany
December 15, 2015
Finnish Centre of Excellence in Inverse Problems Research

http://wiki.helsinki.fi/display/inverse/Home
This is a joint work with

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Esa Niemi, University of Helsinki, Finland

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Outline

Total variation regularized tomography

Industrial case study: low-dose dental imaging

A multiresolution parameter choice method for TV

How about theory?

Bonus material: Comparison to the S-curve method

Bonus material 2: my new lab in Helsinki
Construction of the sinogram
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Unknown: $f \in \mathbb{R}^{32 \times 32}$

Data: $Af \in \mathbb{R}^{49 \times 32}$
The Singular Value Decomposition $A = UDV^T$ allows analysis of any linear inverse problem.
These phantoms have almost the same sinogram
Naive reconstruction using the Moore-Penrose pseudoinverse; data has 0.1% relative noise

Original phantom, values between zero (black) and one (white)  
Naive reconstruction with minimum \(-14.9\) and maximum 18.5
Inverse problem of X-ray tomography: given noisy sinogram, find a stable approximation to $f$

Model space $X = \mathbb{R}^{32 \times 32}$

Data space $Y = \mathbb{R}^{32 \times 49}$

$D(A)$

$A$
We need a regularization strategy involving a suitable family $\Gamma_\alpha : Y \rightarrow X$ of continuous maps.
Constrained Tikhonov regularization

$$\arg \min_{f \in \mathbb{R}^n_+} \left\{ \| Af - m \|_2^2 + \alpha \| f \|_2^2 \right\}$$

Original phantom

Reconstruction

Relative square norm error 35%
Rudin, Osher and Fatemi (1992): total variation regularization arg min \( \min_{f \in \mathbb{R}_+^n} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\} \)

Original phantom

TV regularized reconstruction
Relative square norm error 32%
Naive reconstruction using the Moore-Penrose pseudoinverse

Original phantom

Naive reconstruction

Relative square norm error 1246%
This is Professor Keijo Hämäläinen’s X-ray lab
We collected X-ray projection data of a walnut from 1200 directions.

Laboratory and data collection by Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää.
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

FBP with comprehensive data (1200 projections)  
FBP with sparse data (20 projections)
Sparse-data reconstruction of the walnut using non-negative Tikhonov regularization

Filtered back-projection

Constrained Tikhonov regularization

\[
\arg \min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|f\|_2^2 \right\}
\]
Sparse-data reconstruction of the walnut using non-negative total variation regularization

Filtered back-projection

Constrained TV regularization

$$\arg\min_{f \in \mathbb{R}_+^n} \{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \}$$
TV tomography: \( \arg \min_{f \in \mathbb{R}^n} \{ \| Af - m \|_2^2 + \alpha \| \nabla f \|_1 \} \)

1992 Rudin, Osher & Fatemi: denoise images by taking \( A = I \)
1998 Delaney & Bresler
2001 Persson, Bone & Elmqvist
2003 Kolehmainen, S, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä & Somersalo (first TV work with measured X-ray data)
2006 Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke
2006 Sidky, Kao & Pan
2008 Liao & Sapiro
2008 Sidky & Pan
2008 Herman & Davidi
2009 Tang, Nett & Chen
2009 Duan, Zhang, Xing, Chen & Cheng
2010 Bian, Han, Sidky, Cao, Lu, Zhou & Pan
2011 Jensen, Jørgensen, Hansen & Jensen
2011 Tian, Jia, Yuan, Pan & Jiang
2012–present: dozens of articles indicated by Google Scholar
V. Kolehmainen and I in 2002 after 11 hours of measurements at Instrumentarium Imaging lab
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Bonus material 2: my new lab in Helsinki
The VT device was developed in 2001–2012 by
Lauri Harhanen
Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
Application: dental implant planning, where a missing tooth is replaced with an implant.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics. A panoramic dental image offers a general overview showing all teeth and other dento-maxillofacial structures simultaneously. Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
This is the classical imaging procedure of the panoramic X-ray device
Panoramic dental imaging shows all the teeth simultaneously

Panoramic imaging was invented by Yrjö Veli Paatero in the 1950's.
We reprogram the panoramic X-ray device so that it collects projection data by scanning.
We reprogram the panoramic X-ray device so that it collects projection data by scanning

Number of projection images: 11
Angle of view: 40 degrees
Image size: $1000 \times 1000$ pixels

The unknown vector $f$ has 7,000,000 elements.
Here are example images of an actual patient: navigation image (left) and desired slice (right).


The radiation dose of the VT device is lowest among 3D dental imaging modalities.

<table>
<thead>
<tr>
<th>Modality</th>
<th>$\mu$Sv</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CT</td>
<td>2100</td>
</tr>
<tr>
<td>CB Mercuray</td>
<td>558</td>
</tr>
<tr>
<td>i-Cat</td>
<td>193</td>
</tr>
<tr>
<td>NewTom 3G</td>
<td>59</td>
</tr>
<tr>
<td>VT device</td>
<td>13</td>
</tr>
</tbody>
</table>

[Ludlow, Davies-Ludlow, Brooks & Howerton 2006]

The VT device has been available in the international market since 2008.
Here the CBCT reconstruction (right) gave 100 times more radiation than VT imaging (middle).

Images from the PhD thesis of Martti Kalke.
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Bonus material 2: my new lab in Helsinki
How to choose the regularization parameter in the total variation (TV) functional?

Rullgård 2008

Balancing $\ell^1$ and TV: Clason, Jin & Kunisch 2010

Local variance estimation: Dong, Hintermüller & Rincon-Camacho 2011

Discrepancy principle: Wen & Chan 2012

S-curve method: Kolehmainen, Lassas, Niinimäki & S 2012

Quasi-optimality principle and Hanke-Raus rules: Kindermann, Mutimbu & Resmerita 2013

Including TV parameter in a Karush-Kuhn-Tucker system: Chen, Loli Piccolomini & Zama 2014

Cross validation, Stein’s unbiased risk estimates, L-curve method, ... 

Practical experience suggests that no single choice rule works perfectly for all applications. Therefore, it is good to have a collection of rules.
The continuous tomographic model needs to be approximated using a discrete model

**Continuous model:**

In this schematic setup we have 5 projection directions and a 10-pixel detector. Therefore, the number of data points is 50.

**Discrete model:**
The resolution of the discrete model can be freely chosen according to computational resources

Continuous model:

Discrete models:

In this schematic setup we have 5 projection directions and a 10-pixel detector. Therefore, the number of data points is 50.

The number of degrees of freedom in the three discrete models below are 16, 64 and 256, respectively.
**Intuition:** discrete TV reconstructions at different resolutions should converge to a continuous limit
**Intuition:** discrete TV reconstructions at different resolutions should converge to a continuous limit.
Intuition: discrete TV reconstructions at different resolutions should converge to a continuous limit
We define the total variation (TV) norm consistently for continuous and discrete cases

Continuous anisotropic TV norm for attenuation coefficient $f : \Omega \to \mathbb{R}$:

$$
\int_\Omega \left( |\frac{\partial f}{\partial x_1}| + |\frac{\partial f}{\partial x_2}| \right) dx.
$$

Discrete anisotropic TV norm for an image matrix of size $n \times n$:

$$
\frac{1}{n} \sum |f_{\kappa} - f_{\kappa'}|,
$$
where the sum is over horizontally and vertically neighboring pixel values $f_{\kappa}$ and $f_{\kappa'}$.

The above is based on this approximate two-dimensional computation:

$$
\int_\Omega \left| \frac{f(x_1 + \frac{1}{n}, x_2) - f(x_1, x_2)}{1/n} \right| dx \approx (1/n)^2 \sum \left| \frac{f_{\kappa} - f_{\kappa'}}{1/n} \right|,
$$
where the sum is over horizontally neighboring pixel values $f_{\kappa}$ and $f_{\kappa'}$. 
Low-noise TV reconstructions of a walnut using several regularization parameters

\[ \alpha = 0.001 \quad \alpha = 1 \quad \alpha = 1000 \]

Too small \( \alpha \)  \quad Just right \( \alpha \)  \quad Too large \( \alpha \)

Computations by Kati Niinimäki using a primal-dual interior point method.
Low-noise TV reconstructions of a walnut using several regularization parameters

\( \alpha = 0.001 \) \hspace{2cm} \( \alpha = 1 \) \hspace{2cm} \( \alpha = 1000 \)

Too small \( \alpha \) \hspace{2cm} Just right \( \alpha \) \hspace{2cm} Too large \( \alpha \)

What happens when we compare reconstructions at different resolutions?
Low-noise TV reconstructions of a walnut at many resolutions using $\alpha = 0.001$
Low-noise TV reconstructions of a walnut at many resolutions using $\alpha = 1$
Low-noise TV reconstructions of a walnut at many resolutions using $\alpha = 1000$

128 × 128  192 × 192  256 × 256
**TV norms of low-noise reconstructions with various resolutions and parameters $\alpha$**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>128 × 128</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>1.51</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1.51</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>1.50</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>1.43</td>
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<tr>
<td>$10^0$</td>
<td>1.08</td>
</tr>
<tr>
<td>$10^1$</td>
<td>0.78</td>
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<tr>
<td>$10^2$</td>
<td>0.48</td>
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<tr>
<td>$10^3$</td>
<td>0.12</td>
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<tr>
<td>$10^4$</td>
<td>0.04</td>
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<tr>
<td>$10^5$</td>
<td>0</td>
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<tr>
<td>$10^6$</td>
<td>0</td>
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</tbody>
</table>
### TV norms of low-noise reconstructions with various resolutions and parameters $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Resolution</th>
<th>128 × 128</th>
<th>192 × 192</th>
<th>256 × 256</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>1.51</td>
<td>2.29</td>
<td>3.64</td>
<td></td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1.51</td>
<td>2.29</td>
<td>3.46</td>
<td></td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>1.50</td>
<td>2.23</td>
<td>2.97</td>
<td></td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>1.43</td>
<td>1.85</td>
<td>1.93</td>
<td></td>
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<td>$10^{0}$</td>
<td>1.08</td>
<td>1.11</td>
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<td>$10^{4}$</td>
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<td>$10^{6}$</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

What happens when we add noise to the data?
5% noise TV reconstructions of a walnut at many resolutions using $\alpha = 0.001$
5% noise TV reconstructions of a walnut at many resolutions using $\alpha = 10$
5% noise TV reconstructions of a walnut at many resolutions using $\alpha = 10000$
### TV norms of reconstructions using various noise levels, resolutions and parameters $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Low noise</th>
<th>5% noise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>128$^2$</td>
<td>192$^2$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>1.51</td>
<td>2.29</td>
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<td>1.50</td>
<td>2.23</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>1.43</td>
<td>1.85</td>
</tr>
<tr>
<td>$10^0$</td>
<td>1.08</td>
<td>1.11</td>
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<tr>
<td>$10^1$</td>
<td>0.78</td>
<td>0.78</td>
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<tr>
<td>$10^2$</td>
<td>0.48</td>
<td>0.48</td>
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<tr>
<td>$10^3$</td>
<td>0.12</td>
<td>0.12</td>
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<tr>
<td>$10^4$</td>
<td>0.04</td>
<td>0.04</td>
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<tr>
<td>$10^5$</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$10^6$</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

[Niinimäki, Lassas, Hämäläinen, Kallonen, Kolehmainen, Niemi & S, submitted]
We present another real-data example involving an arrangement of 10 sugarcubes.

We use 120 projections. Shown above is a reconstruction using FBP.
TV norms of low-noise reconstructions with various resolutions and parameters $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Resolution</th>
<th>128 × 128</th>
<th>256 × 256</th>
<th>512 × 512</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td></td>
<td>0.85</td>
<td>2.30</td>
<td>7.40</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td></td>
<td>0.84</td>
<td>2.30</td>
<td>6.80</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td></td>
<td>0.93</td>
<td>2.20</td>
<td>7.00</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td></td>
<td>0.91</td>
<td>1.80</td>
<td>3.10</td>
</tr>
<tr>
<td>$10^{0}$</td>
<td></td>
<td>0.76</td>
<td>0.91</td>
<td>1.00</td>
</tr>
<tr>
<td>$10^{1}$</td>
<td></td>
<td>0.53</td>
<td>0.54</td>
<td>0.55</td>
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<tr>
<td>$10^{2}$</td>
<td></td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
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<tr>
<td>$10^{3}$</td>
<td></td>
<td>0.27</td>
<td>0.30</td>
<td>0.51</td>
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<tr>
<td>$10^{4}$</td>
<td></td>
<td>0.17</td>
<td>0.14</td>
<td>0.09</td>
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<tr>
<td>$10^{5}$</td>
<td></td>
<td>0.11</td>
<td>0.33</td>
<td>1.10</td>
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<tr>
<td>$10^{6}$</td>
<td></td>
<td>0.14</td>
<td>0.28</td>
<td>1.90</td>
</tr>
</tbody>
</table>
TV reconstruction using rounded absolute-value function and projected Barzilai-Borwein

We used 120 projections, parameter $\alpha = 10$, and resolution $512 \times 512$. Data collected by Aki Kallonen, computations by Esa Niemi.
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Bonus material 2: my new lab in Helsinki
Assumptions on the linear forward map $\mathcal{A}$

Assume either (A) or (B) about the linear operator $\mathcal{A}$:

(A) $\mathcal{A} : L^2(D) \to L^2(\Omega)$ is compact and $\mathcal{A} : L^1(D) \to \mathcal{D}'(\Omega)$ is continuous with some open and bounded set $\Omega \subset \mathbb{R}^2$. This covers the case of classical Radon transform with image domain $D$ and sinogram domain $\Omega$. We denote the set of distributions by $\mathcal{D}'(\Omega)$.

(B) $\mathcal{A} : L^1(D) \to \mathbb{R}^M$ is bounded. This covers the practically important discrete pencil beam model of tomographic measurements.
Definition of discrete and continuous regularization functionals

Let $D$ be the square $[0, 1]^2 \subset \mathbb{R}^2$. Use anisotropic $BV(D)$ norm

$$
\|u\|_{BV} = \|u\|_{L^1} + V(u) = \|u\|_{L^1} + \int_D \left( \left| \frac{\partial u(x)}{\partial x_1} \right| + \left| \frac{\partial u(x)}{\partial x_2} \right| \right) dx.
$$

Define $S_\infty : BV(D) \to \mathbb{R}$ and $S_j : BV(D) \to \mathbb{R} \cup \{\infty\}$ by

$$
S_\infty(u) = \|Au - m\|_{L^2(\Omega)}^2 + \alpha_1 \|u\|_{L^1(D)} + \alpha V(u)
$$

with positive regularization parameters $\alpha_1 > 0$ and $\alpha > 0$, and

$$
S_j(u) = \begin{cases} 
S_\infty(u), & \text{for } u \in \text{Range}(T_j), \\
\infty, & \text{for } u \notin \text{Range}(T_j).
\end{cases}
$$

Linear operator $T_j$ projects to functions that are piecewise constant on a regular $2^j \times 2^j$ square pixel grid.
Our main theorem ensures the convergence of regularized solutions as resolution grows

- There exists a minimizer $\tilde{u}_j \in \arg\min(S_j)$ for all $j = 1, 2, 3, \ldots$
- There exists a minimizer $\tilde{u}_\infty \in \arg\min(S_\infty)$.
- Any sequence $\tilde{u}_j \in \arg\min(S_j)$ of minimizers has a subsequence $\tilde{u}_{j(\ell)}$ that converges weakly in $BV(D)$ to some limit $w \in BV(D)$. Furthermore, $\lim_{\ell \to \infty} V(\tilde{u}_{j(\ell)}) = V(w)$.
- The limit $w$ is a minimizer: $w \in \arg\min(S_\infty)$.

[Niinimäki, Lassas, Hämäläinen, Kallonen, Kolehmainen, Niemi & S, submitted]
There are some related results in the literature

1992 Vainikko: *On the discretization and regularization of ill-posed problems with noncompact operators*

We use geometric arguments similar to those here:


These works consider TV functionals and $\Gamma$-convergence when discretization is refined, but without a measurement operator:

2009 Chambolle, Giacomini & Lussardi
2012 Gennip & Bertozzi
2013 Bellettini, Chambolle & Goldman
2013 Trillos & Slepčev

This paper achieves a result analogous with ours, using wavelet frames in the finite-dimensional functionals:

2012 Cai, Dong, Osher & Shen
How to prove the main theorem?

The proof is an analysis of Γ-convergence of functionals $S_j$ to $S_\infty$. However, the choice of topologies is very delicate. For details, see http://arxiv.org/abs/0902.2313v1.

The approximation lemma on the right serves as the foundation of the proof. Generalizations of the result to higher dimension or to other TV norms would require modifying this key lemma.

**Lemma.** For all $u \in BV(D)$ and $\varepsilon > 0$ there exists $j > 0$ and a function $u'$, piecewise constant in the dyadic $2^j \times 2^j$ grid, such that

$$\|u - u'\|_{L^1(D)} + |V(u) - V(u')| < \varepsilon.$$ 

Recall that

$$V(u) = \int_D \left( \left| \frac{\partial u(x)}{\partial x_1} \right| + \left| \frac{\partial u(x)}{\partial x_2} \right| \right) dx.$$
We need to move from triangulation-based to pixel-based approximation

[Bělík and Luskin 2003]: the desired inequality holds with PW constant functions in a fine triangularization. However, we need to work with dyadic $2^j \times 2^j$ pixel grids.
We surround any triangle vertex (blue dot) with a “pixel cluster” neighborhood (gray box)
Refine the grid outside clusters so that pixel-wise polygonal chains (on pink) connect the clusters.
Using the anisotropic BV norm reduces the approximation to estimating small intervals

\[ v_1 = a \]
\[ v_1 = b \]

The difference between the BV norms of the piecewise constant functions \( v_1 \) and \( v_2 \) comes entirely from jumps over the two red vertical intervals below.

\[ v_2 = a \]
\[ v_2 = b \]
What can we say about the proposed method?

Benefits of our multiresolution TV parameter choice method:

▶ simple definition,
▶ easy implementation, and
▶ no need of *a priori* information about the noise amplitude.

Also, it seems to perform well for real tomographic data.

Downside: several reconstructions need to be computed. Also, it is still unclear why the method works so nicely: if there is convergence for any $\alpha$ in theory, what is the instability we are observing?

The method can be tried out with 3D tomography (it works!) and with other inverse problems and regularizers.
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- Total variation regularized tomography
- Industrial case study: low-dose dental imaging
- A multiresolution parameter choice method for TV
- How about theory?

**Bonus material:** Comparison to the S-curve method

**Bonus material 2:** my new lab in Helsinki
We took photographs of walnuts cut in half

These photos are used for estimating the expected total variation norm (number of nonzero gradient elements) in a two-dimensional tomographic reconstruction. We compute the discrete TV norm for these images downsampled to $256 \times 256$ pixel resolution.

[Hämäläinen, Kallonen, Kolehmainen, Lassas, Niinimäki & S 2013]

Special thanks to Esa Niemi for his careful job in sawing the walnuts.
The S-curve method applied to the real data case with no added noise. (Multiresolution gave $\alpha = 1$)

$\alpha = 3$

$0.76$

$10^{-2}$

$256 \times 256$

[Niinimäki, Lassas, Hämäläinen, Kallonen, Kolehmainen, Niemi & S, submitted]
The S-curve method applied to the real data case with 5% noise. (Multiresolution gave $\alpha = 10$)

$S$-curve

$\alpha = 40$

$256 \times 256$

[Niinimäki, Lassas, Hämäläinen, Kallonen, Kolehmainen, Niemi & S, submitted]
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This is my new X-ray laboratory at University of Helsinki
Lego robot under imaging
Lego robot under imaging
This is simply a SIRT reconstruction from 360 views using the ASTRA toolbox

Computation and visualization by Topias Rusanen
Thank you for your attention!

http://www.siltanen-research.net
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All Matlab codes freely available at this site!