Low-dose three-dimensional X-ray imaging

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https://wiki.helsinki.fi/display/inverse/Home
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Background of low-dose 3D X-ray imaging

The first encouraging experiments

Bayesian inversion for 3D X-ray imaging

VT: a novel dental implant planning device

Further developments: wavelets, Besov spaces and 4D
Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography.

Nobel prize was awarded to Hounsfield (top) and Cormack in 1979.
Reconstruction of a function from its line integrals was first invented by Johann Radon (1887-1956).

This is the famous inversion formula from 1917 for the Radon transform $Rf$ of a function $f$:

$$f(x) = \frac{1}{4\pi^2} \int_{S^1} \int_{\mathcal{R}} \frac{d}{ds}(Rf)(\theta, s) \frac{x \cdot \theta - s}{dsd\theta} dx$$
Filtered back-projection (FBP) is mathematical technology used on a daily basis in hospitals around the world. The quality of 3D reconstruction using FBP is excellent.

However, a comprehensive data set is mandatory for FBP.
Due to the radiation dose, a CT scan is only appropriate for seriously ill patients.

In filtered backprojection, the mathematical reconstruction formula assumes dense angular sampling of full-angle data: the chosen mathematics requires high radiation dose.

Think the opposite: take as few X-ray images as possible and use tailor-made mathematics to form a reconstruction that is good enough for the clinical task at hand. Then the low level of radiation dose requires new mathematics and more computational power.
A series of projects started in 2001 aiming for a new type of low-dose 3D imaging

The goal was a mathematical algorithm with

**Input:** small number of digital X-ray images taken with any X-ray device

**Output:** three-dimensional reconstruction with high enough quality for the clinical task at hand

Products of Instrumentarium Imaging in 2001:
Tuned Aperture Computed Tomography (TACT) was the starting point of our research

Instrumentarium Corp. licensed Richard Webber’s tomosynthesis-based TACT patent in 1999.

Tomosynthesis can be dated back to the work of Ziedses des Plantes in 1932.

EINE NEUE METHODE ZUR DIFFERENZIERUNG IN DER RÖNTGENOGRAPHIE (PLANIGRAPHIE)¹

von

B. G. Ziedses des Plantes
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Detector

X-ray source positions

Tooth donated to science by Helena Sarlin.
The projection images look like this:

![Projection Images](image-url)
Horizontal slices:

- **truth**
  - full angle

- **Bayes**
  - limited angle

- **tomo**
  - limited angle
Some parts of the boundary are strongly visible in projection data

Microlocal analysis of recoverable singularities is available in Greenleaf & Uhlmann (1989), Quinto (1993) and Ramm & Katsevich (1996), based on earlier work by Guillemin.
Vertical slices:

Experimental setup for chairside 3D imaging models the clinical situation.
Details of this limited angle experiment

Opening angle 60 degrees

Seven digital intraoral radiographs
(664 x 872 pixels each)

There are 42,496,000 unknowns and 4,053,056 linear equations. Computation is divided into 400 approximately 2D problems.
Tomo-synthesis

Bayes-MAP

(singularities visible as analysed in Quinto 1993 and Ramm & Katsevich 1996)

S, Kolehmainen, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä and Somersalo 2003
Data from a knee phantom using a surgical C-arm X-ray device, reconstruction by level set method

Kolehmainen, Lassas & S (2008), US patent 7274766
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Direct problem of tomography is to find the radiographs from given tissue
Inverse problem of tomography is to find the tissue from radiographs.

9 unknowns, 11 linear equations.
Inverse problem of tomography is to find the tissue from radiographs.
The limited angle problem is harder than the full angle problem.

- 9 unknowns, 11 linear equations
- 9 unknowns, 6 linear equations
In limited angle 3D imaging there are many tissues matching the radiographs. 

\[
\begin{array}{ccc}
8\sqrt{2} & & \\
9\sqrt{2} & 4 & 4 & 5 & 13 \\
1\sqrt{2} & 1 & 3 & 4 & 8 & \\
& 1 & 0 & 2 & 3 & \\
\end{array}
\]

\[
\begin{array}{ccc}
5 & 6 & 2 \\
1 & 5 & 2 \\
4 & 0 & -1 & \\
9 & 1 & 3 \\
1 & 0 & 7 \\
3 & 0 & 0 & \\
\end{array}
\]

*a priori* information is needed!
We write the reconstruction problem in matrix form and assume Gaussian noise.

Our measurement is $m = Ax + \varepsilon$ with Gaussian noise $\varepsilon$ of standard deviation $\sigma > 0$. 

Construct system matrix $A$ so that $Ax = m$. 

$x = [x_1, x_2, \ldots, x_9]^T$

$m = [m_1, m_2, \ldots, m_6]^T$
Bayes formula combines measured data and *a priori* information together

We reconstruct the most probable 3D tissue in light of
1. Available radiographs and
2. Physiological *a priori* information

Bayes formula gives the *posterior distribution* $p(x|m)$:

$$p(x|m) \sim p(x)p(m|x)$$

- **Prior distribution**, or tissue model
- **Likelihood distribution**, or measurement model

$$p(m|x) = p_\varepsilon(Ax-m) \sim \exp\left(-\frac{1}{2\sigma^2}\|Ax - m\|_2^2\right)$$
Bayesian inversion algorithms are flexible and widely applicable

Algorithms can be tailored to any measurement geometry.

Naturally modular software: measurement model (likelihood) and tissue model (prior) can be designed independently.

Estimating $x$ leads to large-scale optimization or to integration in high-dimensional space:

Maximum a posteriori (MAP) estimate:

$$ x_{\text{MAP}} = \arg \max_x p(x|m). $$

Conditional mean (CM) estimate:

$$ x_{\text{CM}} = \int_{\mathbb{R}^n} x p(x|m) \, dx. $$
We build a prior distribution for dental tissue using total variation prior

Positivity constraint:

\[ p_+(x) = \begin{cases} 
0 & \text{if } x_j < 0 \text{ for some } j \\
1 & \text{otherwise}
\end{cases} \]

Approximate total variation penalty:

\[ p_{TV}(x) = \exp(-\alpha \sum_{\text{neighbors}} |x_\ell - x_k| \beta) \]

\[ |t|_\beta = \frac{1}{\beta} \log(\cosh(\beta t)) \]
**Computation of the MAP estimate**

Large scale optimization problem:

\[
x_{\text{MAP}} = \arg \min_{x_j \geq 0} \left\{ \frac{1}{2\sigma^2} \| Ax - m \|^2_2 + \alpha \sum_{\text{neighbors}} |x_\ell - x_k|_\beta \right\}
\]

We use the gradient method of Barzilai & Borwein, which is a modification of Euler’s steepest descent method.

\[
\alpha_j = \frac{(x_j - x_{j-1}) \cdot (\nabla \Phi(x_j) - \nabla \Phi(x_{j-1}))}{(x_j - x_{j-1}) \cdot (x_j - x_{j-1})}
\]

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**VT: a novel dental implant planning device**

Further developments: wavelets, Besov spaces and 4D
Application: dental implant planning, where a missing tooth is replaced by an implant
Three-dimensional information is crucial for dental implant planning

The hole must be drilled deep enough for sturdy attachment but not so deep that the mandibular nerve is damaged.

Two-dimensional X-ray projection images are not suitable for assessing the proper depth because of geometric distortion.

Three-dimensional reconstruction of the tissue is needed, but a traditional CT scan is not practical due to high cost, too low resolution and too high radiation dose.

Low-dose 3D X-ray imaging is an ideal solution: it is based on a small number of X-ray projection images recorded with a cost-effective device.
Panoramic X-ray device rotates around the head and produces a general picture of teeth

Panoramic imaging was invented by Yrjö Paatero in 1950’s.

Nowadays a panoramic device is standard equipment at every dental clinic around the world.

In our project, we reprogrammed the device so that it collects limited-angle data.
We reprogram the panoramic X-ray device so that it collects projection data by scanning

11 projection images of the mandibular area

40 degrees angle of view

1000 x 1000 pixels per image, formed by a scanning movement
Limited angle Bayesian reconstruction can be used for locating the mandibular nerve

7 200 000 unknowns, 2 100 000 data points. Parallel processing with a 13-node Beowulf cluster and GPGPU computing yields computation time less than 4 minutes.

Cederlund, Kalke & Welander (2009)
Kolehmainen, Lassas & S (2008)
United States patent 7269241
The Bayesian low-dose imaging technique has been commercialized by Palodex Group

The **VT device** has been in the market from year 2007; thousands of units sold. See [www.vt-cube.com](http://www.vt-cube.com).

It is remarkable that an existing 2D X-ray imaging product (panoramic device) becomes a 3D imaging product by a software update.

The core of that update is a mathematical inversion algorithm.
The radiation dose of the VT device is the lowest among 3D dental imaging modalities

<table>
<thead>
<tr>
<th>Modality</th>
<th>μSv</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head CT</td>
<td>2100</td>
</tr>
<tr>
<td>CB Mercuray</td>
<td>558</td>
</tr>
<tr>
<td>i-Cat</td>
<td>193</td>
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<tr>
<td>NewTom 3G</td>
<td>59</td>
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<tr>
<td>VT device</td>
<td>12</td>
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<tr>
<td>Panoramic image (2D)</td>
<td>6</td>
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</tbody>
</table>

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Wavelet transform divides a function into details at different scales
We introduce a convenient renumbering of the basis functions 

$$f(x) = \sum_{\ell=1}^{\infty} c_{\ell} \psi_{\ell}(x)$$
Besov space norms can be written in terms of wavelet coefficients:

The function

\[ f(x) = \sum_{\ell=1}^{\infty} c_\ell \psi_\ell(x) \]

belongs to \( B^s_{pq}(\mathbb{T}^d) \) if and only if

\[ 2^j s 2^{dj} \left( \sum_{\ell=2^jd}^{2(j+1)d-1} |c_\ell|^p \right)^{1/p} \in \ell^q(\mathbb{N}). \]

In particular, \( f \in B^1_{11}(\mathbb{T}^2) \) if and only if

\[ \sum_{\ell=1}^{\infty} |c_\ell| < \infty. \]
Computation of the CM estimate reduces to sampling from well-known densities

\(B_{11}^{1}(\mathbb{T}^{2})\) prior: write \(U\) in wavelet basis as

\[
U = \sum_{\ell=1}^{\infty} X_{\ell} \psi_{\ell}
\]

with each \(X_{\ell}\) distributed independently \(\sim \exp(-|x|)\).

Posterior distribution of \(U_{n}\) takes the following form in terms of wavelet coefficients \(x_{1}, \ldots, x_{n}\):

\[
C \exp \left( -\frac{1}{2} \| M_{k}(\omega_{0}) - A \sum_{\ell=1}^{n} x_{\ell} \psi_{\ell}\|_{L^{2}(\mathbb{T}^{2})}^{2} - \alpha \sum_{\ell=1}^{n} |x_{\ell}| \right)
\]

Direct and inverse wavelet transforms are easy and quick to compute.
Limited angle tomography results for X-ray mammography

Rantala et al. (2006)
US patent 7215730

MAP estimate, Besov prior, \( p=1.5=q \) and \( s=0.5 \)
Local tomography results for dental X-ray imaging; data from dry mandible (jawbone)

Niinimäki, S & Kolehmainen (2007); Vänskä, Lassas & S (2009); US patent 7215730
Comparison of our local tomography results with Lambda-tomography

Lambda-tomography

Besov prior, $p=q=1.5$ and $s=0.5$
Number of data is $k$, number of unknowns is $n$.
Number of data is $k$, number of unknowns is $n$
Number of data is $k$, number of unknowns is $n$
Number of data is $k$, number of unknowns is $n$

$k=16$

$n=440$
Number of data is $k$, number of unknowns is $n$

$k=24$
$n=440$
In particular, we have studied edge-preserving Bayesian inversion

**Counterexample** (Lassas and S 2004) Bayesian inversion using **Total variation prior** is not discretization-invariant.

**Theorem** (Lassas, Saksman and S 2008) Bayesian inversion using $B^{\frac{1}{11}}_{11}(\mathbb{T}^2)$ **Besov prior** is discretization-invariant.
Low-dose three-dimensional X-ray imaging based on a small number of projection images is feasible for many diagnostic and operational tasks.

The incompleteness of such sparse data needs to be compensated by different mathematics and heavier computation than that involved in filtered back-projection.

More details available at www.siltanen-research.net