Three-dimensional X-ray Vision by Sparse Tomography

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Finland

UNIVERSITY OF JYVÄSKYLÄ

TAMPERE UNIVERSITY OF TECHNOLOGY

FINNISH METEOROLOGICAL INSTITUTE

AALTO UNIVERSITY

University of Eastern Finland

Lappeenranta University of Technology

University of Helsinki
Outline

Traditional X-ray tomography

Tomography apart from X-rays

Tomographic imaging with sparse data

Industrial case study: low-dose 3D dental X-ray imaging

Current research topics
  Sparse-angle tomography
  Tomography for moving objects
  Color tomography

Conclusion
This is a tomographic slice through a human head. How are images like this created?
The rate of X-ray attenuation varies inside the head because there are different tissues.

https://youtu.be/lvUAOeS1sv8
After calibration we are observing how much attenuating matter the X-ray encounters in total.

https://youtu.be/RFArLtWEfsQ
This sweeping movement is the data collection mode of first-generation CT scanners

https://youtu.be/JHUz5oyeZb0
Modern CT scanners look like this
Modern scanners rotate at high speed

https://commons.wikimedia.org/wiki/File:CT-Rotation.ogv
This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAqI
This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58IO1FKw
How does filtered back-projection work?
The inverse problem of tomography is to recover the unknown target from the measured X-ray data.

https://youtu.be/YhClb0MaB70
Summing all the back-projections results in a blurred reconstruction.
Summing all the back-projections results in a blurred reconstruction.
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Summing all the back-projections results in a blurred reconstruction.
Summing all the back-projections results in a blurred reconstruction
Here we use more directions, so the reconstruction quality is higher

https://youtu.be/5DUGTXd26nA
Final reconstruction involves filtering on top of the back-projection.

\[ \hat{f}(\xi) \]

Multiplication with “ice-cream cone”

\[ |\xi| \hat{f}(\xi) \]

FFT

IFFT
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Electron transmission cryotomography reveals the swimming engine of *Treponema primitia* bacteria

[Murphy, Leadbetter & Jensen 2016]
Cosmic muon imaging revealed a secret chamber inside the Pyramid of Cheops
Imaging with neutrons opens up new possibilities as water attenuates but metal is transparent

Video:
Anders Kaestner
Neutron Imaging and Activation Group, Paul Scherrer Institute
The mathematics of X-ray tomography can be used for recovering the ozone layer.

European Space Agency
Finnish Meteorological Institute
Envisat and GOMOS projects
Thanks to Johanna Tamminen!
Tomography appears in adaptive optics

- Modern telescope imaging suffers from turbulence in the atmosphere ⇒ blurring of images
- **Adaptive optics** corrects the perturbed incoming light in real-time
- Major challenge in wide-field AO: atmospheric tomography

Helin, Kindermann, Lehtonen & Ramlau 2018
Yudytskiy, Helin & Ramlau 2014

European Extremely Large Telescope (2024)
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We collected X-ray projection data of a walnut from 1200 directions

Laboratory and data collection by Keijo Hämäläinen and Aki Kallonen, performed at University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

- FBP with comprehensive data (1200 projections)
- FBP with sparse data (20 projections)
Sparse-data reconstruction of the walnut using non-negative total variation regularization

\[
\arg\min_{f \in \mathbb{R}^n_+} \left\{ \| Af - m \|_2^2 + \alpha \| \nabla f \|_1 \right\}
\]
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Application: dental implant planning, where a missing tooth is replaced with an implant
This is the classical imaging procedure of the panoramic X-ray device

https://www.youtube.com/watch?v=QFTXegPxC4U
The resulting image shows a sharp layer positioned inside the dental arc.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics.

A panoramic dental image offers a general overview showing all teeth and other structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
We reprogram the panoramic X-ray device so that it collects projection data by scanning

https://www.youtube.com/watch?v=motthjiP8ZQ
We reprogram the panoramic X-ray device so that it collects projection data by scanning

Number of projection images: 11
Angle of view: 40 degrees
Image size: $1000 \times 1000$ pixels

The unknown vector $f$ has 7,000,000 elements.
Standard Cone Beam CT reconstruction delivers 100 times more radiation than VT imaging.

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke 2006
Kolehmainen, Lassas & S 2008
Cederlund, Kalke & Welander 2009
Hyvönen, Kalke, Lassas, Setälä & S 2010
U.S. patent 7269241, thousands of VT units in use
The VT device was developed in 2001–2012 by

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
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Conclusion
Normal Knee

Osteoarthritis

Image by Bruce Blaus, CC BY-SA 4.0
https://commons.wikimedia.org/w/index.php?curid=44968165
Faster measurement by recording fewer projections

FBP, 40 angles

Shearlet-sparsity, 40 angles

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Dynamic tomography with optical flow constraint

https://youtu.be/CV7CClQJy2M

Burger, Dirks, Frerking, Hauptmann, Helin & S 2017
“Atmospheric mathematics” project: tracking fluid and gas transport in plants

Joint work with Tatiana Bubba, Tiia Grömholm, Hanna Help, Simo Huotari, Alexander Meaney, Timo Vesala & Anne Wald
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Measurement geometry

```
kV
Target
Rotation
direction
X-ray detector
```

$\text{kV}_1$
Measurement geometry
Measurement geometry
Measurement geometry
Multi-energy CT Measurements:
GE Nanotom 180F at Dept. of Physics, UH

- X-ray detector
- 50 µm pixel size
- Frozen bird phantom
- Sample manipulator
- X-ray tube
- W target
Comparison of methods: material decomposition into water and bone bases

<table>
<thead>
<tr>
<th>Method</th>
<th>Projections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground truth</td>
<td>2160</td>
</tr>
<tr>
<td>No prior</td>
<td>90</td>
</tr>
<tr>
<td>Our method</td>
<td>90</td>
</tr>
</tbody>
</table>

[Kolehmainen, Meaney, S, Toivanen, unpublished]
With Prof. Huotari, we will construct a novel multi-source microtomography system in Helsinki.
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Automatic regularization parameter selection: controlled wavelet domain sparsity

February 5, 2018

Summary: Choosing the regularization parameter is a hard problem, for which many approaches exist. Here we discuss the recently developed and fully automatic method, called controlled wavelet domain sparsity (CWDS) in the context of X-ray tomography. This approach involves sparsity promoting regularization with respect to an orthogonal wavelet basis. The (more...)

The D-bar Method for Electrical Impedance Tomography – Experimental Data

January 18, 2018

Summary: Electrical Impedance Tomography (EIT) aims to measure the internal electrical conductivity of a physical body from electro-
Thank you for your attention!
Links to open computational resources

Open CT datasets:
- Finnish Inverse Problems Society (FIPS) dataset page

Matrix-based parallel-beam reconstruction algorithms: FIPS Computational Blog
- Truncated SVD
- Total Variation regularization

Matrix-free large-scale reconstruction algorithms:
- Matlab page of Mueller-S 2012 book
- ASTRA toolbox
- TVReg: Software for 3D Total Variation Regularization
Part I: Linear Inverse Problems
1 Introduction
2 Naïve reconstructions and inverse crimes
3 Ill-Posedness in Inverse Problems
4 Truncated singular value decomposition
5 Tikhonov regularization
6 Total variation regularization
7 Besov space regularization using wavelets
8 Discretization-invariance
9 Practical X-ray tomography with limited data
10 Projects

Part II: Nonlinear Inverse Problems
11 Nonlinear inversion
12 Electrical impedance tomography
13 Simulation of noisy EIT data
14 Complex geometrical optics solutions
15 A regularized D-bar method for direct EIT
16 Other direct solution methods for EIT
17 Projects
Another great resource is Per Christian Hansen’s 3D tomography toolbox TVreg

**TVReg**: Software for 3D Total Variation Regularization (for Matlab Version 7.5 or later), developed by Tobias Lindstrøm Jensen, Jakob Heide Jørgensen, Per Christian Hansen, and Søren Holdt Jensen.

Website: http://www2.imm.dtu.dk/ pcha/TVReg/
These books are recommended for learning the mathematics of practical X-ray tomography

1983 Deans: The Radon Transform and Some of Its Applications
1986 Natterer: The mathematics of computerized tomography
1988 Kak & Slaney: Principles of computerized tomographic imaging
1996 Engl, Hanke & Neubauer: Regularization of inverse problems
1998 Hansen: Rank-deficient and discrete ill-posed problems
2001 Natterer & Wübbeling: Mathematical Methods in Image Reconstruction
2008 Buzug: Computed Tomography: From Photon Statistics to Modern Cone-Beam CT
2008 Epstein: Introduction to the mathematics of medical imaging
2010 Hansen: Discrete inverse problems
2012 Mueller & S: Linear and Nonlinear Inverse Problems with Practical Applications
2014 Kuchment: The Radon Transform and Medical Imaging