Tomographic imaging of moving objects: a space-time approach

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http://wiki.helsinki.fi/display/inverse/Home
Outline

Background: low-dose dental 3D imaging

X-ray tomography for moving objects
  A spacetime level set method
  Reconstructions with (2+1)D simulated data
  Reconstructions with measured data

Future directions
Application: dental implant planning, where a missing tooth is replaced with an implant.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics.

A panoramic dental image offers a general overview showing all teeth and other dento-maxillofacial structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
This is the classical imaging procedure of the panoramic X-ray device
Panoramic dental imaging was originally invented at University of Helsinki in the 1950’s.

The VT device was developed in 2001–2012 by
Lauri Harhanen
Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
(Note: SFINX 2015–2017)
We reprogram the panoramic X-ray device so that it collects projection data by scanning
We reprogrammed the panoramic X-ray device so that it collects projection data by scanning.

Number of projection images: 11
Angle of view: 40 degrees
Image size: $1000 \times 1000$ pixels
The unknown vector $f$ has 7,000,000 elements.
Here are example images of an actual patient: navigation image (left) and desired slice (right).


The radiation dose of the VT device is lowest among 3D dental imaging modalities.

<table>
<thead>
<tr>
<th>Modality</th>
<th>$\mu$Sv</th>
</tr>
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<tbody>
<tr>
<td>Head CT</td>
<td>2100</td>
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<tr>
<td>CB Mercuray</td>
<td>558</td>
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<td><strong>VT device</strong></td>
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[Ludlow, Davies-Ludlow, Brooks & Howerton 2006]

The VT device has been available in the international market since 2008.
Here the CBCT reconstruction (right) gave 100 times more radiation than VT imaging (middle).

Images from the PhD thesis of Martti Kalke.
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Future directions
This is a joint work with

Keijo Hämäläinen, University of Helsinki, Finland

Lauri Harhanen, Technical University of Denmark

Aki Kallonen, University of Helsinki, Finland

Ville Kolehmainen, University of Eastern Finland

Matti Lassas, University of Helsinki, Finland

Esa Niemi, University of Helsinki, Finland
We study a tomographic imaging modality based on multiple source-detector pairs

Place several X-ray sources and detectors in fixed positions in 3D. The detectors should have a high frame-rate relative to the movement of the object under imaging.

Reconstructing the 3D structure at all frames leads to 4D tomography.

Applications include cardiac imaging, angiography, biotechnology research, veterinary medicine, nondestructive testing.
One potential benefit of this imaging modality is three-dimensional angiography.

This is regular two-dimensional angiography.
Video by Dr. Magda Bayoumi, downloaded from Dailymotion
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The level set method can be used to model mud
The level set method [Osher, Sethian] parametrizes curves and surfaces in a flexible way.
\[ H(\phi) \]

\[ g(\phi) \]
A generalization of the classical level set method was introduced in [Kolehmainen, Lassas & S 2008]

We model the X-ray attenuation function as $g(\Phi(x, y))$, where

$$g(\tau) = \begin{cases} 
\tau, & \text{if } \tau \geq 0 \\
0, & \text{if } \tau < 0.
\end{cases}$$

The smooth level set function $\Phi(x, y) := \lim_{s \to \infty} \phi(x, y, s)$ is the large-time limit of the solution of the evolution equation

$$\begin{cases} 
\phi_s = -A^*(A(g(\phi)) - m) + \beta \Delta \phi, \\
(\nu \cdot \nabla - r)\phi|_{\partial \Omega} = 0,
\end{cases}$$

with a suitable initial condition.

Here $\beta > 0$, $r \geq 0$, $A^*$ denotes the transpose of $A$, and $\Delta \phi = \phi_{xx} + \phi_{yy}$. 
The generalized level set method works nicely for stationary limited-angle 2D tomography.

Images from [Kolehmainen, Lassas & S 2008]. However, see also [Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
We deal with the dynamic case by considering the moving target in spacetime
We write a higher-order level set method in (2+1)D spacetime for the dynamic case

The 2D static case:

We model the X-ray attenuation as $g(\phi(x, y))$. The level set function $\phi$ belongs to $H^1(\Omega)$ and is defined as the minimizer of

$$\|Ag(\phi) - m\|_{L^2}^2 + \alpha\|\nabla\phi\|_{L^2}^2,$$

where $\nabla\phi = [\phi_x, \phi_y]^T$.

[Kolehmainen, Lassas & S 2008]

The (2+1)D dynamic case:

We model the X-ray attenuation as $g(\Phi(x, y, t))$. The level set function $\phi$ belongs to $H^2(\Omega \times [0, T])$ and is defined as the minimizer of

$$\|Ag(\phi) - m\|_{L^2}^2 + \alpha\|\nabla\phi\|_{L^2}^2 + \alpha(\|\partial_t^2\phi\|_{L^2}^2 + \|\partial_y^2\phi\|_{L^2}^2 + \|\partial_t^2\phi\|_{L^2}^2),$$

where $\nabla\phi = [\phi_x, \phi_y, \phi_t]^T$.

[Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
There exists at least one minimizer for our higher-order functional

**Theorem**: Let $\mathcal{A}$ be an operator modeling 2D Radon transforms measured at several times. Then the functional

$$F_n(\phi) := \frac{1}{2} \| \mathcal{A}g(\phi) - m \|^2_2 + \frac{\alpha}{2} \sum_{1 \leq |\beta| \leq n} \| D^\beta \phi \|^2_2,$$

has a **global minimizer**. The minimizer is unique for $n = 1$.

[Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
Numerical minimization in the case $n = 2$

We smooth out the nondifferentiability of the objective functional by replacing $g : \mathbb{R} \to \mathbb{R}$ by the differentiable approximation

$$g_\delta(\tau) = \begin{cases} \sqrt{\tau^2 + \delta^2} - \delta, & \text{if } \tau > 0, \\ 0, & \text{if } \tau \leq 0, \end{cases}$$

where $\delta > 0$ is small.

Now we can use a gradient-based optimization method for computing the minimizer of

$$\|A g_\delta(\phi) - m\|_{L^2}^2 + \alpha \|\nabla \phi\|_{L^2}^2 + \alpha (\|\partial_x^2 \phi\|_{L^2}^2 + \|\partial_y^2 \phi\|_{L^2}^2 + \|\partial_t^2 \phi\|_{L^2}^2).$$
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Future directions
Two simulated examples, based on only seven (7) projection directions:

Imaging geometry:

Spacetime phantoms:
Level set reconstruction in spacetime

Original phantom
Original  | Level set  | Level set  | Tikhonov  | TV  \\
|-------|-------|-------|---------|-------\\n|       | \(n = 1\) | \(n = 2\) | 2D, \(\geq 0\) | 2D, \(\geq 0\) \\

[Niemi, Lassas, Kallonen, Harhanen, Hämäläinen and S 2015]
Level set reconstruction in spacetime

Original phantom
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X-ray sources and detectors are expensive, and currently we have only one of each.

How to use one X-ray source and one detector to create a multi-source type dataset?
Consider a simple multi-source measurement:
Consider a simple multi-source measurement:

\[ t = 2 \]
Consider a simple multi-source measurement:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

$$t = 1$$
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 1 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 1 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation: \[ t = 2 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 2 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation: $t = 2$
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
We can collect the same dataset by rotating, if the object stays stationary during rotation:

\[ t = 3 \]
Level set \( n = 2 \) reconstruction from 10 projections

\[ t = 1 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections
Level set ($n = 2$) reconstruction from 10 projections

$\mathbf{t} = 2$

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections

Level set ($n = 2$) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 3 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 4 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 5 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 6 \]

**Tomographic data:**
Keijo Hämäläinen
Aki Kallonen

**Reconstruction:**
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
FBP reconstruction from 120 projections

\( t = 7 \)

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
Level set \((n = 2)\) reconstruction from 10 projections

\[ t = 8 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

FBP reconstruction from 120 projections
Level set ($n = 2$) reconstruction from 10 projections

FBP reconstruction from 120 projections

$t = 9$

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set ($n = 2$) reconstruction from 10 projections
FBP reconstruction from 120 projections

\[ t = 10 \]

Tomographic data:
Keijo Hämäläinen
Aki Kallonen
Reconstruction:
Esa Niemi

Level set \((n = 2)\) reconstruction from 10 projections
This is a movie showing the recovered level set in (2+1) dimensional spacetime

Computation and visualization by Esa Niemi
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This is my new X-ray laboratory at University of Helsinki
The next step: 4D imaging of moving objects

Data and video: thanks to Alexander Meaney and Topias Rusanen
Lego robot under imaging
Lego robot under imaging
This is simply a SIRT reconstruction from 360 views using the ASTRA toolbox

Computation and visualization by Topias Rusanen
The next reconstructions have been done by

Hendrik Dirks, University of Münster, Germany

Lena Frerking, University of Münster, Germany

Andreas Hauptmann, University of Helsinki, Finland
Thank you for your attention!

http://www.siltanen-research.net
All Matlab codes freely available at this site!

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6 Total variation regularization
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Part II: Nonlinear Inverse Problems
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12 Electrical impedance tomography
13 Simulation of noisy EIT data
14 Complex geometrical optics solutions
15 A regularized D-bar method for direct EIT
16 Other direct solution methods for EIT
17 Projects
A series of projects started in Finland in 2001, aiming for a new type of low-dose 3D imaging. The goal was a mathematical algorithm with

**Input:** small number of digital X-ray images from any X-ray device.  
**Output:** Three-dimensional reconstruction with high enough quality for the clinical task at hand.

Products of Instrumentarium Imaging in 2001:
Experimental setup for chairside 3D imaging models the clinical situation
Details of this limited-angle experiment

We recorded seven intraoral images of pixel size $664 \times 872$ with $60^\circ$ total angle of view.

There are $42\,496\,000$ unknowns and $4\,053\,056$ linear equations. The computation is divided into 400 two-dimensional problems.
Tomo-synthesis

S, Kolehmainen, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä and Somersalo 2003