The magic of math: X-ray vision

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The Maths of Biology
Stockholm, Sweden, October 10, 2018
This my industrial-academic background

2009: Professor, University of Helsinki, Finland

2006: Professor, Tampere University of Technology, Finland

2005: R&D scientist at Palodex Group

2004: R&D scientist at GE Healthcare

2002: Postdoc at Gunma University, Japan

2000: R&D scientist at Instrumentarium Imaging

1999: PhD, Helsinki University of Technology, Finland
Outline

What is an X-ray image?

Slice imaging: X-ray tomography

Are you a natural tomographer?

Filtered back-projection (FBP)

Industrial case study: low-dose 3D dental X-ray imaging

X-ray vision with small number of X-rays

X-ray vision without X-rays
Wilhelm Conrad Röntgen invented X-rays and was awarded the first Nobel Prize in Physics in 1901.
X-ray intensity attenuates inside matter, here shown with a homogeneous block

https://www.youtube.com/watch?v=lfXo2S1xXCQ
X-ray images are very useful for doctors. For example, they can see fractures.
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Here is a 2D slice through a human head

Andrew Ciscel, Wikimedia commons
Now the attenuation process is more complicated because there are different tissues

https://youtu.be/lvUAOeS1sv8
After calibration we are observing how much attenuating matter the X-ray encounters in total

https://youtu.be/RFArLtWEfsQ
This sweeping movement is the data collection mode of first-generation CT scanners

https://youtu.be/JHUz5oyeZb0
Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography.

Hounsfield (top) and Cormack received Nobel prizes in 1979.
Modern CT scanners look like this
Modern scanners rotate at high speed

https://commons.wikimedia.org/wiki/File:CT-Rotation.ogv
This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAql
This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58lO1FKw
Diagnosing stroke with X-ray tomography

Ischemic stroke

CT image from Jansen 2008

Hemorrhagic stroke

CT image from Nakano et al. 2001
Contrast-enhanced CT of abdomen, showing liver metastases
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Let’s warm up before the tests.
Here is tomographic data of a simple object:
Can you guess the shape of the object from the tomographic data?
Test 1: can you guess the image?
Test 1: is the shape (a), (b) or (c)?
Test 1: the image is (c): triangle.
Test 2: can you guess the image?

https://youtu.be/YhClb0MaB70
Test 2: is the shape (a), (b) or (c)?
Test 2: the image is (b): two discs.
Test 3: can you guess the image?

https://youtu.be/
Test 3: is the shape (a), (b) or (c)?
Test 3: the image is (b): character B.
Test 4: can you guess the image?

https://youtu.be/
Test 4: is the image (a), (b) or (c)?
Test 4: the image is (c): unicorn emoji.
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The inverse problem of tomography is to recover the unknown target from the measured X-ray data.

https://youtu.be/YhClb0MaB70
Since we know the projection directions, we can back-project the data into the image.
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Back-projection becomes more useful by summing up the images
Summing all the back-projections results in a blurred reconstruction.
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Here we use more directions, so the reconstruction quality is higher

https://youtu.be/5DUGTXd26nA
Final reconstruction involves filtering on top of the back-projection.

\[ \hat{f}(\xi) \]

Multiplication with “ice-cream cone”

\[ |\xi| \hat{f}(\xi) \]

FFT \[ \rightarrow \]

IFFT
This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58IO1FKw
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Application: dental implant planning, where a missing tooth is replaced with an implant.
Dental X-ray imaging 100 years ago
It is tedious to interpret a mosaic of overlapping intraoral X-ray images
Panoramic dental imaging shows all the teeth simultaneously

Panoramic imaging was invented by Yrjö Veli Paatero in the 1950’s.
This is the classical imaging procedure of the panoramic X-ray device

https://www.youtube.com/watch?v=QFTXegPxC4U
The resulting image shows a sharp layer positioned inside the dental arc.
Nowadays, a digital panoramic imaging device is standard equipment at dental clinics.

A panoramic dental image offers a general overview showing all teeth and other structures simultaneously.

Panoramic images are not suitable for dental implant planning because of unavoidable geometric distortion.
We reprogram the panoramic X-ray device so that it collects projection data by scanning

https://www.youtube.com/watch?v=motthjiP8ZQ
We reprogram the panoramic X-ray device so that it collects projection data by scanning

Number of projection images: 11

Angle of view: 40 degrees

Image size: 1000×1000 pixels

The unknown vector $f$ has 7 000 000 elements.
Standard Cone Beam CT reconstruction delivers 100 times more radiation than VT imaging

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke 2006
Kolehmainen, Lassas & S 2008
Cederlund, Kalke & Welander 2009
Hyvönen, Kalke, Lassas, Setälä & S 2010
U.S. patent 7269241, thousands of VT units in use
The VT device was developed in 2001–2012 by

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L. Webber
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We collected X-ray projection data of a walnut from 1200 directions.
Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

FBP with comprehensive data (1200 projections)

FBP with sparse data (20 projections)
Sparse-data reconstruction of the walnut using non-negative total variation regularization

Filtered back-projection

Constrained TV regularization

\[
\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}
\]
A projection image is produced by parallel X-rays and several detector pixels (here three pixels)

\[
\begin{array}{ccc}
4 & 4 & 5 \\
1 & 3 & 4 \\
1 & 0 & 2 \\
\end{array}
\]

- 13 \(=4+4+5\)
- 8 \(=1+3+4\)
- 3 \(=1+0+2\)
For tomographic imaging it is essential to record projection images from different directions.
The length of X-rays traveling inside each pixel is important, thus here the square roots

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The direct problem of tomography is to find the projection images from known tissue.
The inverse problem of tomography is to reconstruct the interior from X-ray data.
The limited-angle problem is harder than the full-angle problem

9 unknowns, 6 equations

9 unknowns, 11 equations
In limited-angle imaging, different objects may produce the same data.
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Tomography appears in adaptive optics

- Modern telescope imaging suffers from turbulence in the atmosphere ⇒ blurring of images
- Adaptive optics corrects the perturbed incoming light in real-time
- Major challenge in wide-field AO: atmospheric tomography

European Extremely Large Telescope (2024)

Helin, Kindermann, Lehtonen & Ramlau 2018
Yudytskiy, Helin & Ramlau 2014
The mathematics of X-ray tomography can be used for recovering the ozone layer.

European Space Agency
Finnish Meteorological Institute
Envisat and GOMOS projects
Thanks to Johanna Tamminen!
Cosmic muon imaging revealed a secret chamber inside the Pyramid of Cheops
Imaging with neutrons opens up new possibilities as water attenuates but metal is transparent.

Video:
Anders Kaestner
Neutron Imaging and Activation Group,
Paul Scherrer Institute
Electron transmission cryotomography reveals the swimming engine of *Treponema primitia* bacteria

[Murphy, Leadbetter & Jensen 2016]
Thank you for your attention!